Privacy, Payments and Financial Stability *

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Abstract

I study how lenders' access to the information in borrowers' payment data affects financial stability and welfare. When lenders can infer more about borrower quality, they are able to discontinue investment projects with low pledgeable returns. Doing so harms borrowers whose projects have high non-pledgeable returns. By offering privacy, a central bank digital currency (CBDC) would facilitate risk-sharing between borrowers and lenders. At the same time, however, a private means of payment like CBDC will affect equilibrium interest rates and banks' portfolio choices. I show that privacy leads banks to hold more liquid asset portfolios and thereby increases financial stability. In equilibrium, borrowers with non-pledgeable returns benefit from privacy in payments, while lenders are worse off. The central bank can balance some of these effects by providing information back to lenders.

Keywords: Privacy, Anonymous Payment, Central Bank Digital Currency, Bank Stability

JEL Classification: E22, E61, G21, G28

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1 Introduction

Payment data generates valuable information and brings privacy concerns. Consumers' payment histories reveal their preferences (Garratt and Van Oordt (2021)), and banks can use their digital footprints to predict their default risk (Berg et al. (2020), Parlour et al. (2022)). Borrowers may also be concerned about their private information being used against their interests. For example, entrepreneurs who borrow from banks may worry about their loans not being renewed based on information from payment data. As advancing computer technology makes extracting information from payment data easier and leaves privacy more vulnerable, the privacy issue in payments becomes more urgent. Such privacy concerns create a potential role for a more anonymous form of payment that keeps the users' information private. In response, many central banks are actively considering offering central bank digital currency (CBDC), which could fulfill such privacy needs. Compared with cash, it could be widely and conveniently used for transactions in large amounts. The payment information would also accrue to the central bank, which has no incentive for making profits. As Lagarde (2018) stated:

"This (central bank digital) currency could satisfy public policy goals, such as (i) financial inclusion, (ii) security and consumer protection; and to provide what the private sector cannot: (iii) ***privacy in payments****."*[1]

However, anonymous forms of payment may adversely affect banks. As cash usage has declined, banks handle the majority of payment activities in the economy. These activities create information that is useful for monitoring loans and managing assets. Bringing in another widely used payment option like CBDC may reduce the information available in the current banking system, a concern expressed in the European Central Bank (ECB)'s report:

"If banks decrease their role in deposit-taking and intervene less in the routing of payment instructions, they might have less information about clients, which, in turn, would harm their risk assessment capacity."[2]

Despite this concern, the macroeconomic implications of such information loss are unclear. In this paper, I show how a privacy-preserving payment option, which serves as an outside option for borrowers and reduces banks' ability to acquire information, impacts the lending market through bank's portfolio choices, financial stability, and welfare.

In this paper, I construct a model in which a group of entrepreneurs have heterogeneous incentives for privacy. The traditional informative payment option (which I call 'debit card

[1] In IMF speech, *Winds of Change: The Case for New Digital Currency*, 2018
[2] European Central Bank (2020)

payment') cannot keep their production information private from the bank. I then introduce a new form of payment which I call CBDC. This new form of payment does not immediately reveal the entrepreneurs' production information to the bank. I show that introducing CBDC facilitates risk-sharing between entrepreneurs and the bank: Entrepreneurs with high privacy incentives would switch to CBDC, leaving the bank with less information about these borrowers and bearing more risk. The increased uncertainty changes the bank's desired composition of assets between loans and liquid reserves. In particular, the higher risk in lending to borrowers who use the anonymous payment method (CBDC) leads banks to charge them a higher interest rate and results in lower returns to the banks in equilibrium. As a result, the bank holds a more liquid portfolio, which results in a lower probability of experiencing a liquidity shortage. Thus, the banks are *more stable*. In terms of welfare, introducing CBDC generates a welfare tradeoff between borrowers and lenders. Depositors' welfare decreases due to lower investment returns. Entrepreneurs' welfare increases, even for those who value privacy less and continue to reveal information to banks.

I connect the bank's asset and liability sides to analyze the impact of anonymity. On its liability side, the bank faces aggregate uncertainty in liquidity demand by depositors. On its liability side, the bank makes a portfolio choice that balances the possibility of a liquidity shortage against the higher investment return, in the spirit of Champ et al. (1996) and the more recent work of Kim and Kwon (2023). What is more, I introduce private information and risky investments on the bank's asset side. Banks lend to borrowers (entrepreneurs) who demand funds to make production, and there are two means of payment for entrepreneurs to choose from. Entrepreneurs have private information about their production activities and the incentive to keep such information private: They generate non-pledgeable benefits if their investments are funded without being interrupted by the bank. This feature is similar to Aghion and Bolton (1992), in which the borrowers value some important variables that cannot be verified or written in the contract. For example, if the production is terminated halfway, it hurts the entrepreneur's reputation; or the entrepreneur values the experience of producing even though the project fails eventually. On the other hand, competitive banks rely on the payment information to better monitor the loans. They can observe the return before the investment matures if the entrepreneurs use the debit card payment. They maximize depositors' utilities and can terminate unprofitable projects to lower lending risks and avoid costly management. In the absence of CBDC, banks that have limited commitment can fully observe the loan returns and only continue to fund profitable loans, but at the cost of borrowers' privacy gains.

When the anonymous CBDC is introduced, entrepreneurs can choose between the debit card and CBDC payment options in their production activities. The optimal choice depends critically on loan rates and on the entrepreneur's privacy gains. CBDC serves as a commitment device that allows the borrowers to preserve privacy gains. However, this causes information loss for banks, which cannot acquire the production information in time: Banks cannot observe the CBDC users' investment return before it matures. In addition, this may contribute to a higher management cost for banks. This management cost, which relies on the aggregate information available in the banking system, can be thought of as detecting borrowers' misbehavior. This assumption captures the ECB's concern that anonymity may deteriorate bank assets. As a result, the CBDC payment users' loan rate will be higher. This result holds even when this management cost is the same for both payment options. In equilibrium, only entrepreneurs with high privacy incentives will use CBDC, while others stick to the informative debit payment. The information loss from anonymous payment facilitates risk-sharing with additional management costs for banks. It leads to a decrease in aggregate lending and a lower investment return at the equilibrium, in which both payment options are active. Banks optimally chooses a more liquid investment portfolio, which eventually leads to banks' less likely to experience liquidity shortages. Thus, banks are more stable despite being less informative about the loans in time. In addition, the debit payment users' loan rate decreases as the returns from both payment users should be equal at the equilibrium. Compared to no CBDC case, entrepreneurs who do not value privacy much can also benefit from the introduction of the anonymous CBDC payment, due to the decreased loan rate when using the traditional debit payment method.

Lastly, I discuss the potential role of the central bank in dealing with information accrued in the CBDC payment option. The central bank issues digital currency and naturally has the advantage of accumulating information. The CBDC loan rate can be a potential policy tool for the central bank that could indirectly influence banks' portfolio choice and stability. The central bank could also reduce borrowing costs by lowering banks' management costs. However, this may not unambiguously improve the welfare of borrowers because the loan rates interact and can move in the opposite direction.

Literature Review Many recent studies focus on privacy and the use of customers' data. For example, Garratt and Van Oordt (2021) study how the customers value privacy because they would face less price discrimination. Lee and Garratt (2021) show that payment competition leads to the data monopolist, while anonymity preserves the competitive market structure and improves customers' welfare. In another paper, Kang (2021) shows multiple equilibria exist when the seller can profit from purchasing the consumers' private information and predicting their preferences. In all these papers, individuals favor anonymity for privacy concerns since they worry their information is used against their benefit, either by the transaction's counterparty or the third party. There is a similar concern in my model. If it acquires the information early, the bank values the entrepreneurs' private information and can take advantage of it. On the other hand, the entrepreneurs have privacy incentive that conflicts with the bank. This structure is similar to Aghion and Bolton (1992) and Hart and Moore (1998), who discussed the incomplete contract between a penniless entrepreneur and a wealthy investor. In my model, whether banks can or cannot acquire the information to stop funding unprofitable projects depends on the means of payment options. This difference gives rise to the different loan rates for entrepreneurs using these two payment options.

There is also a fast-growing literature on CBDC's potential impact on monetary policy and financial system from different perspectives. For example, the welfare effect of CBDC competing with cash and credit like Monnet et al. (2021) and Keister and Sanches (2023); the crowding in effect when banks have market power Chiu et al. (2023). For banking stability, there is a concern for CBDC's disintermediation effect that it may crowd out bank deposits like in Piazzesi and Schneider (2020), Bindseil (2020) and Gross and Schiller (2021). CBDC may also provide flight-to-safety and trigger bank runs as in Williamson (2021). Fernández-Villaverde et al. (2021) discuss the potential of having a central bank deposit monopolist, deterring bank runs. In addition, Keister and Monnet (2020) discussed the information perspective of CBDC on bank stability. In their work, the account-based CBDC provides additional information to the central bank so that bank runs can be detected and intervened early. Compared to the stability research above, I focus on an anonymous CBDC's potential impact from information loss. The anonymous payment diverts information away from banks, resulting in a new perspective in a bank's portfolio choice: information loss contributes to choosing a more liquid portfolio when anonymous and informative payment options coexist.

The risk-sharing mechanism is also related to the 'Hirshleifer effect' as Hirshleifer (1971) and more recent works like Andolfatto et al. (2014) and Izumi (2021). In Izumi (2021), opacity in banks' asset values provides insurance to depositors that mitigates run incentives. Andolfatto et al. (2014) discuss the regulator's optimal disclosure policy about asset quality in markets. Nondisclosure facilitates risk-sharing when the regulator lacks commitment to reveal asset quality information. In my work, it is the competitive banks that lack commitment and prevent risk-sharing that could be socially desirable. The anonymous CBDC payment option serves as a commitment device that brings the risk-sharing back by delaying information acquisition. More importantly, I show the information loss's impact on stability: The bank responds by holding a more liquid portfolio so the banks do not become more unstable.

The rest of the paper is organized as follows. In section 2, I describe the model, show the borrower and bank's problems, and discuss the socially optimal solution. In section 3, I show the equilibrium without CBDC, in which the representative bank does not value borrowers' privacy gains. Then in section 4, I introduce the CBDC payment option and show that the fraction of CBDC borrowers is endogenously determined. I compare the stability and welfare with and without anonymous CBDC. Section 5 discusses the central bank's potential role in mitigating banks' information loss. Section 6 concludes the findings.

2 The Model

2.1 Entrepreneurs

There are three periods $t = 0, 1, 2$. At period 0, a continuum of penniless entrepreneurs with measure one needs funds to produce. All entrepreneurs consume at $t = 2$ and have the same production technology: For each unit of good invested at $t = 0$, the investment generates R units with probability δ at $t = 2$; and 0 otherwise. An individual entrepreneur i chooses the production quantity q^i under a linear disutility $L(q^i) = -q^i$. The entrepreneur consumes c_e^i and enjoys some non-transferable private benefit b^i from production. The entrepreneur i has the following quasi-linear utility function:

$$u(c_e^i, q^i) = 2\sqrt{c_e^i} - q^i + \mathbb{I}^i \cdot b^i$$

Where $\mathbb{I}^i \in \{0, 1\}$ is the indicator for acquiring the private benefit. In particular, the entrepreneur gets b^i only if the production activity is successfully funded through and matures at $t = 2$ ($\mathbb{I}^i = 1$). If the production is terminated at $t = 1$, there will be no gains ($\mathbb{I}^i = 0$). This non-transferable benefit provides the incentive for privacy, i.e., to keep the investment information private during production. For simplicity, I assume the private benefit is uniformly distributed across all borrowers as $b^i \sim U(0, 1)$.

The entrepreneur's production process involves transactions such as paying for workers and purchasing raw materials. The entrepreneur can 'learn by doing' and discover his project's return privately at $t = 1$ based on information underneath these transactions. For example, a car manufacturer may realize the profit at $t = 2$ would be low if the bills for operating the auto assembly line (paying for workers, buying machines) remain high during production at $t = 1$. In this economy, the entrepreneur chooses a payment option that determines whether the lending bank can or cannot acquire the payment information during production. This decision choice is similar to He et al. (2023) in which the borrowers own the data and can decide whether to share it with lenders. In particular, an entrepreneur can choose the informative "debit card" payment issued by the bank, enabling the bank to observe the payment activities and thus the investment's return at $t = 1$. Alternatively, he can use the anonymous CBDC, so the bank cannot observe the return until the investment matures at $t = 2$. Depending on his choice, the entrepreneur thus faces two different loan rates: the anonymous CBDC loan rate ρ_0 (CBDC rate hereafter) and the informative debit card payment loan rate ρ_1 (debit rate hereafter).

At period 0, an entrepreneur i chooses his production level q^i under the given loan rates ρ_0, ρ_1. If the entrepreneur chooses to use debit card payment, he accepts ρ_1 and produces q_1^i using the debit card, which allows the lending bank to observe his payment flows. In this sense, there is no information asymmetry as the lending bank can acquire the same information as the entrepreneur. Thus, the bank can also realize the investment return when it is known by the entrepreneur in period 1. Importantly, the bank *may* terminate the loan, i.e., $\mathbb{I}^i = 0$, when the bank observes the investment is unprofitable. The entrepreneur rationalizes this, knowing that the bank may terminate the unprofitable investments. It is easy to see if the project's future return will be good (with probability δ) the project is worth continuing. However, if the project's future will be bad, the bank may choose $\mathbb{I}^i = 0$. So, we can write the entrepreneur i's problem as:

$$\max_{q_1^i} \quad \delta \cdot 2\sqrt{(R-\rho_1) \cdot q_1^i} - q_1^i + \delta b^i + (1-\delta)\mathbb{I}^i b^i$$

The FOC gives his demand for the loan as:

$$q_1^i = \delta^2 \cdot (R - \rho_1) \tag{1}$$

His demand depends on the given loan rate ρ_n, and the private benefit b^i does not affect his loan demand here. Note this equation also provides the aggregate demand for loan since there is one unit of entrepreneurs in total. In addition, the entrepreneur's indirect utility has two parts: the consumption from production after paying back the loan and the private benefit. The entrepreneur i's utility can be expressed as:

$$\Pi_1^i = \delta^2 \cdot (R - \rho_1) + \delta b^i + (1-\delta)\mathbb{I}^i b^i \tag{2}$$

Similarly, the entrepreneur can choose the anonymous CBDC payment. If he chooses CBDC, the entrepreneur i ensures his private benefit b^i. The bank cannot observe the production's return at $t = 1$ since it cannot analyze the payment flows of the entrepreneurs. The entrepreneur i's problem in this case can be written as:

$$\max_{q_0^i} \quad \delta \cdot 2\sqrt{(R-\rho_0) \cdot q_0^i} - q_0^i + b^i.$$

Compared with no CBDC case, the anonymous payment ensures the entrepreneur's private gain b^i. The entrepreneur's demand for the loan is:

$$q_0^i = \delta^2 \cdot (R - \rho_0) \tag{3}$$

His indirect utility as the function of the CBDC loan rate is:

$$\Pi_0^i = \delta^2 \cdot (R - \rho_0) + b^i \tag{4}$$

Importantly, an entrepreneur's payment choice depends on his indirect utility Π_0^i and Π_1^i, which are affected by the privacy gain b^i. I will show in section 3 there is a threshold $\bar{b}$ such that entrepreneurs who have high private gains with $b^i > \bar{b}$ will use anonymous CBDC, while those with $b^i < \bar{b}$ will continue to use the debit payment.

2.2 Depositors and the Representative Bank

At $t = 0$, there is also a continuum of depositors with measure one; each endowed with one unit of the single good and having the logarithmic utility function $ln(c)$. Among the depositors, a random fraction of π becomes impatient and needs consumption at $t = 1$, while the rest are patient and want to consume in period 2. The depositors pool their resources in the bank as in Diamond and Dybvig (1983) at $t = 0$, but the random variable π satisfying $\pi \sim U(0, 1)$ will be publicly realized at the beginning of $t = 1$.

A competitive bank collects funds from depositors and lends them to entrepreneurs at $t = 0$. A representative bank takes the loan rate ρ_n as given. Facing depositors' uncertain liquidity demand, the bank makes a portfolio choice between holding liquid cash reserves and lending to the entrepreneurs whose projects mature at $t = 2$. Denote the offers to impatient and patient depositors as $c_1(\pi)$ and $c_2(\pi)$, respectively. The bank chooses the fraction γ of deposits to hold as a cash reserve and lends the remaining $1 - \gamma$ to entrepreneurs. When CBDC is introduced, the bank also chooses the composition of lending θ, so that fraction $\theta(1 - \gamma)$ lends to debit card entrepreneurs and $(1 - \theta)(1 - \gamma)$ to CBDC entrepreneurs. Since the uncertainty in aggregate liquidity demand only resolves at $t = 1$, the bank may run out of its liquid assets at $t = 1$ when it meets the liquidity demand. In this paper, the bank is defined as unstable when it runs out of cash reserves, i.e., it experiences a liquidity crisis, as in Champ et al. (1996).

In addition, I assume the bank incurs a management cost per unit of investment for continuing the loans at $t = 2$. In particular, I assume the debit card payment's management cost holds as a constant $\bar{k}$. This cost is sufficiently small. The CBDC payment's management cost is given by:

$$k_{dc} = \bar{k} + \beta(1 - \int_0^1 \mathbb{I}^i) \tag{5}$$

In which $1 - \int_0^1 \mathbb{I}^i \in [0,1]$ naturally represents the fraction of CBDC payment users. This expression shows the CBDC payment's management cost increases as a larger fraction of borrowers choose CBDC payment. The interpretation is that it is more costly to manage the less informed loans to prevent moral hazard problems. Less information may hamper the bank's ability to monitor and manage assets in the banking system. Banks can better know whether a project operates well by comparing it to other projects with more information. The bank can detect the borrowers' misbehaving by analyzing and comparing the entrepreneurs' payment flows. On the other hand, less information (a smaller I) on the loans and the aggregate economy contributes to higher management costs. In addition, note that due to management cost k, the bank would want to stop funding the investments that generate negative returns (minus k). The entrepreneurs who use the debit card payment will not capture privacy gain when the investment fails.

Denote the bank's investment return as R_{b0} and R_{b1}, corresponding to the per unit investment return from debit and CBDC borrowing. In particular, the *per unit expected* investment return from debit payment borrowers is:

$$R_{b1} = \delta\rho_0 - \mathbb{I}^i\bar{k} \tag{6}$$

And the per unit investment return from CBDC borrowers is:

$$R_{b0} = \delta\rho_1 - k_{dc} \tag{7}$$

The debit rate R_{b1} depends on the individual bank's termination choice since the bank can choose $\mathbb{I}^i$ when borrower i uses debit payment. Note the choice of $\mathbb{I}^i$ is made after observing the future return, so R_{b1} measures the *expected* returns. On the other hand, the CBDC rate R_{b0} depends on the aggregate level of CBDC borrowing cost k_{dc} and is exogenous from the bank's perspective. This also captures the point that the bank loses information and cannot influence the loan ex-post lending. The return R_{b1} is the expected return since the bank pools the loans together while only a fraction δ of each type generates positive returns. As a result, the bank's problem can be written in the following:

$$
\begin{aligned}
\max_{\gamma,\theta,c_1(\pi),c_2(\pi),\alpha(\pi)} \quad & \int_0^1 [\pi \cdot lnc_1(\pi) + (1-\pi) \cdot lnc_2(\pi)] \cdot f(\pi) d\pi \\
\text{s.t.} \quad & \pi c_1(\pi) = \alpha(\pi)\gamma \\
& (1-\pi)c_2(\pi) \leq \gamma - \alpha(\pi)\gamma + (1-\gamma)[\theta R_{b1} + (1-\theta)R_{b0}] \\
& 0 \leq \alpha(\pi) \leq 1 \\
& 0 \leq \theta \leq 1
\end{aligned}
$$

The choice $\alpha(\pi) \in [0,1]$ represents the fraction of cash the bank uses to pay the early depositors. And the returns R_{b1} and R_{b0} are given by (6) and (7). When there is no CBDC, I can simply impose $\theta = 1$ so all lending is through the debit payment option.

Note the choice $\mathbb{I}^i = \{0, 1\}$ implicitly means the bank has *limited commitment* when lending. In period 1, the bank acquires the production information by observing the debit card payment flows. The bank realizes the investment return as the entrepreneurs. The bank may terminate the loan due to the management costs: If the production return would be R at $t = 2$, the bank would continue funding the investment; otherwise, the bank could terminate the project. The debit card borrowers will also rationalize the bank is able to terminate the project if the project is not profitable at $t = 1$.[3]

The focus of the bank's problem is on the possibility of a liquidity shortage. Note the bank will choose how much to pay depositors *after* observing the aggregate shock π so the payments c_1 and c_2 is contingent on π. The bank may run out of its liquid holdings to pay early depositors. Whether a liquidity shortage occurs depends on whether $\alpha = 1$ holds. In particular, in the appendix I show that there is a threshold $\bar{\pi}_n$ such that if $\pi < \bar{\pi}_n$, the depositors' payoffs satisfy $c_{n1}(\pi) = c_{n2}(\pi) = \gamma_n + (1-\gamma_n)R_n$, and the bank has extra liquid assets ($\alpha < 1$). For $\pi \geq \bar{\pi}_n$, the payoffs are $c_{n1}(\pi) = \frac{\gamma_n}{\pi}$ and $c_{n2}(\pi) = \frac{(1-\gamma_n)R_n}{1-\pi}$, while the bank runs out of reserves with $\alpha = 1$. In these cases, a large amount of impatient depositors (a large π) will share the limited reserves and the bank cannot treat all depositors the same.[4] Thus, the bank experiences a liquidity shortage and is unstable when the realized $\pi > \bar{\pi}_n$.

The optimal choice of cash reserve can be solved from the FOC of γ_n:

$$1 - \gamma_n = \int_{\bar{\pi}_n}^1 F(\pi) d\pi \tag{8}$$

Where $\bar{\pi}_n = \frac{1}{1+(\frac{1}{\gamma_n}-1)R_b}$; and $R_b = \theta R_{b1} + (1-\theta)R_{b0}$ with CBDC, and $R_b = R_{b1}$ without.

[3] To focus on the ex-ante borrowing choices, I assume the means of payment is enforceable ex-post: if the entrepreneur chooses one payment, he immediately use it for transactions in production and cannot switch to the other type after the loan is granted.

[4] When $\pi = \bar{\pi}_n$, the bank chooses exactly $\alpha = 1$ with $c_1 = c_2$ and is defined as no liquidity shortage.

The term $F(.)$ is the cumulative distribution of π. The investment return R_b can be shown greater than one because $\bar{k}$ is relatively small with $\delta R - \bar{k} > 1$. Lastly, note equation (8) also implicitly provides the aggregate supply of funds, $1 - \gamma_n$, as a function of the loan rates ρ_0, ρ_1, depending on with/without CBDC.

2.3 Timeline

The timeline in Figure 1 summarizes the environment above. At $t = 1$, entrepreneurs request funds from the bank. Banks collect from depositors, choose a portfolio of loans and liquid assets, and arrange payments for depositors. The loan rates depend on the entrepreneur's payment options in the production process. The debit card payment allows the bank to see the potential return when the entrepreneur knows it at $t = 1$, while the CBDC payment blocks the bank from observing this information. At the beginning of period 2, the management costs k incurred for continuing loans. Lastly, the entrepreneurs repay the loans, and the patient depositors withdraw.

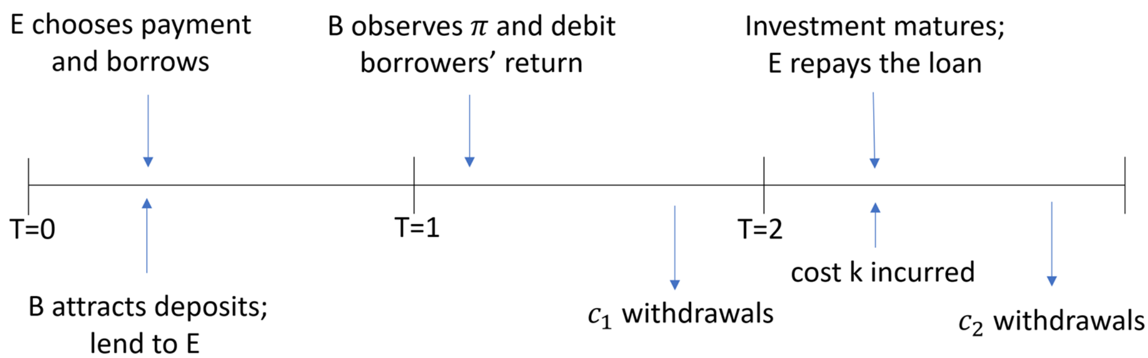

Figure 1. Timeline

2.4 The Socially Optimal Solution

I first discuss the socially optimal solution as a benchmark for welfare comparison later. A planner maximizes the utilities of both entrepreneurs and depositors. It sets some optimal loan rate ρ^* and makes a portfolio choice γ^*. Most importantly, it chooses $\mathbb{I}^i = \{0, 1\}$ for *every* loan to determine whether the project can continue at period 1. The planner's objective function can be written as:

$$\max_{\rho^*, \gamma^*, \mathbb{I}^i} \int_0^1 [\delta^2(R - \rho^*) + \mathbb{I}^i \cdot b^i] db^i + \int_0^1 [\pi \cdot lnc_1(\pi) + (1 - \pi) \cdot lnc_2(\pi)] f(\pi) d\pi$$

Subject to the constraints of the bank's problem.

A project may not be socially desirable even though it generates private gains for the entrepreneur. The tradeoff emerges between borrowers (entrepreneurs) and lenders (depositors). Once the planner observes that an investment will generate zero return in period 2, it chooses whether to continue the project in period 1. On the one hand, the entrepreneurs can receive privacy gains, which they acquire by lasting the projects to period 2. On the other hand, depositors (banks) benefit from terminating unprofitable projects ex-post (i.e., after knowing the future return) and avoiding incurring additional management costs. Consider the marginal entrepreneur j: allowing the project to continue generates marginal gain b^j; the bank generates a marginal loss $\bar{k}$ on investment that transmits to the marginal welfare loss on depositors' expected utility. The tradeoff means it is optimal to let some projects that generate zero return for banks continue from a social perspective. The following proposition states this.

Proposition 1. *There exists a privacy gain level $b^* > 0$ such that for $b^i > b^*$, the entrepreneur's project should always be funded to maturity in period 2. When $k_{dc} = \bar{k}$, $b^* < \bar{k}$.*

The social optimum is intuitive: When the privacy gain is relatively larger than the banks' expected costs, it is socially optimal to generate risk-sharing and let some projects mature for privacy gains even though this incurs costs on banks. What is more, if we drop the assumption that CBDC payment is potentially more costly to monitor, i.e., if we simply assume $k_{dc} = \bar{k}$, then the threshold is below $\bar{k}$ due to the concavity of depositors' utilities. I will use this threshold as an example to help with welfare comparison later.

3 Equilibrium Without CBDC

In this section, I solve the equilibrium in which only the informative debit card payment is available to entrepreneurs. I will characterize the entrepreneur's problem for loan demand. Next, I solve the bank's problem to derive the loan supply. Then I will find the equilibrium loan rate and discuss banks stability as characterized by the probability of liquidity shortage.

3.1 Equilibrium and Bank Liquidity

To begin with, I show that a representative bank, once it observes the future investment return, will always terminate unprofitable projects.

Lemma 1. *The bank always chooses $\mathbb{I}^i = 0$ at $t = 1$, if it observes the project i to generate zero return at $t = 2$.*

This result is intuitive: From the bank's perspective, it is always optimal to terminate unprofitable projects so the return R_b and depositors' utilities are higher. As a result, the per unit return from debit payment in (6) can be simplified to:

$$R_{b1} = \delta\rho_1 - \delta\bar{k} \tag{9}$$

Note the same type of loans are pooled together, and R_{b1} is the per unit investment return: A fraction $1-\delta$ of debit borrowers being terminated with $\mathbb{I}^i = 0$, and the rest δ fraction of projects are worth continuing and incurring cost $\bar{k}$.

Meanwhile, entrepreneurs anticipate this potential termination when borrowing. However, without an outside anonymous option such as CBDC, the bank's limited commitment will not directly influence their borrowing amount, as shown in (1). Equating this demand and the funds supply $1-\gamma$, I have the market clearing condition:

$$1-\gamma = \delta^2(R-\rho_1) \tag{10}$$

In addition, I apply the uniform distribution $F(\pi) = \pi$ and simplify (8) as:

$$1-\gamma = \frac{1}{2} - \frac{1}{2}\left(\frac{1}{1+(\frac{1}{\gamma}-1)(\delta\rho_1-\delta\bar{k})}\right)^2 \tag{11}$$

Denote the equilibrium cash reserve ratio as γ_n^* and the equilibrium loan rate as ρ_n^*. These two conditions above pin down the equilibrium with $\{\gamma_n^*, \rho_n^*\}$. The following proposition states there is a unique equilibrium with interior solutions of γ_n^* and $\bar{\pi}_n$.

Proposition 2. *There is a unique interior equilibrium with $\{\gamma_n^*, \rho_n^*\}$ in the economy. If $\bar{k}$ increases, γ_n^* and $\bar{\pi}_n$ increase with a more liquid bank portfolio.*

Without CBDC, the competitive banking system has a unique equilibrium in which the bank chooses a portfolio to balance the possibility of liquidity shortage against the higher investment return. To see this, We can rewrite $\delta\rho_n^* = \delta R - \frac{1}{\delta}(1-\gamma_n^*)$ and replace $\delta\rho_n^*$ in (11), now I can get the condition that pins down γ_n^*:

$$1-\gamma_n^* = \frac{1}{2} - \frac{1}{2}\left(\frac{1}{1+(\frac{1}{\gamma_n^*}-1)(\delta R - \frac{1}{\delta}(1-\gamma_n^*) - H_n)}\right)^2 \tag{12}$$

In which $H_n = \delta\bar{k}$, capturing the management cost incurs. In the appendix, I show that the right-hand-side (RHS) of (12) is concave and decreasing in γ_n. Note if $\gamma = 1$, the RHS equals 0 with the slope smaller than minus one[5] thus there is only one interior solution. The

[5] Which does not depend on the assumption on the distribution of π.

corner solution, $\gamma = 1$, is not optimal since it means the bank chooses not to lend at all.

The tradeoff in γ_n^* comes from higher investment return R_b and the probability of experiencing liquidity shortage. As can be seen from (8) if the realized liquidity demand π is larger than $\bar{\pi}_n$, the bank is running out of cash reserves and is said to experience a liquidity shortage. The marginal $\bar{\pi}_n^*$ at the equilibrium naturally measures the probability of running out of cash reserve at $t = 1$, which is determined by the γ_n^*. A more liquid investment portfolio leads to a lower $\bar{\pi}_n$ and a more stable bank.

The proposition also shows how the changed management cost can affect bank return and stability. If the bank's loan becomes more costly, the bank optimally chooses a more liquid portfolio because the investment is less attractive. This can be seen from the bank's return, $R_b = \delta R - \frac{1}{\delta}(1 - \gamma_n) - \delta\bar{k}$. For any $k' > \bar{k}$, the bank's return would decrease, which would further contribute to a more liquid portfolio choice with higher cash reserve γ_n^* by the bank.

Note without CBDC, the individual entrepreneur's privacy gain does not affect the equilibrium R_b and the loan rate. Any changes in management costs only comes from profitable projects, as captured by $H_n = \delta\bar{k}$. In the next section, I will show how the introduction of CBDC contributes to information loss by the bank. The anonymous payment will enforce more risk-sharing between the entrepreneurs and the bank. The bank's management cost and return change due to the risk-sharing, as well as its liquidity choice and stability.

4 Equilibrium With CBDC

In this section, I introduce the anonymous CBDC payment that can substitute the informative debit card payment. The anonymous CBDC payment is a commitment tool that ensures projects to mature. The bank must wait until $t = 2$ to acquire information by observing the investment return. The bank's ability to manage the loans is affected, reflecting more risk-sharing and a different CBDC payment management cost k_{dc}, which will also be endogenously determined.

4.1 Equilibrium and Choice of Payments

Firstly, entrepreneurs can choose the anonymous CBDC payment option to make transfers. In this way, the anonymous CBDC payment serves as a commitment tool allowing entrepreneurs to keep the realized return private at $t = 1$. Recall the privacy gain on project i depends on $\mathbb{I}^i$, i.e., the bank's choice to terminate or not, the entrepreneurs can now compare their utility gains between the traditional debit payment and this new privacy-preserving

CBDC payment. In particular, an entrepreneur i compares (2) and (4) as mentioned in section 2. Imposing $\mathbb{I}^i = 0$ for the $1-\delta$ fraction of debit borrowers, we can see for given loan rates, there is a threshold $\bar{b}$ that when $b^i = \bar{b}$, $\Pi_0^i = \Pi_1^i$ holds. The entrepreneurs with privacy gains lower than $\bar{b}$ will continue to choose debit payment; while those who value privacy more ($b^i > \bar{b}$) will opt out to use CBDC payment. The entrepreneur on the margin with $b^i = \bar{b}$ is indifferent between the payment options. This endogenous threshold $\bar{b}$ is determined by the indifference condition $\Pi_0^i = \Pi_1^i$:

$$\delta(\rho_0 - \rho_1) = \frac{(1-\delta)}{\delta} \cdot \bar{b} \tag{13}$$

Secondly, the bank now keeps a fraction θ of funds to those debit payment borrowers while dividing $1-\theta$ of the loans to CBDC payment. At the equilibrium, the bank would be indifferent between the two types of lending with $R_{b0} = R_{b1}$, which can be seen from the first order to θ. This indifference condition can be written as:

$$\delta(\rho_0 - \rho_1) = (1-\delta)\bar{k} + \beta(1-\bar{b}) \tag{14}$$

The first order condition of γ now becomes:

$$1 - \gamma = \int_{\frac{1}{1+(\frac{1}{\gamma}-1)[\theta R_{b1} + (1-\theta) R_{b0}]}}^{1} F(\pi) d\pi \tag{15}$$

This is similar to the case without CBDC except for changes in the expression of the bank's investment return. Both types of lending satisfy the supply equals demand when the market clears.

$$\theta(1-\gamma) = \int_0^{\bar{b}} \delta^2 (R - \rho_1) \cdot f(b^i) db^i \tag{16}$$

$$(1-\theta)(1-\gamma) = \int_{\bar{b}}^{1} [\delta^2 (R - \rho_0)] \cdot f(b^i) db^i \tag{17}$$

The first equation is the clearing condition for debit card payment users, and the second is the clearing condition for anonymous CBDC payment users. These conditions (13) - (17) provide the equilibrium solutions for $\{\gamma^*, \rho_0^*, \rho_1^*, \bar{b}^*, \theta^*\}$. The following proposition shows the existence of a unique equilibrium in which both payment options are actively used.

Proposition 3. *There is a unique interior equilibrium with* $\{\gamma^*, \rho_0^*, \rho_1^*, \bar{b}^*, \theta^*\}$ *that the anonymous CBDC and debit card payments coexist.*

The anonymous payment is attractive to those whose privacy gain is high since it is costly to have their projects terminated by banks. The uniqueness of the equilibrium can be shown by a condition that pins down the equilibrium ρ_1^*, similar to equation 12:

$$1-\gamma^* = \frac{1}{2} - \frac{1}{2}\left(\frac{1}{1+(\frac{1}{\gamma^*}-1)(\delta R - \frac{1}{\delta}(1-\gamma^*) - H_{coe}(\bar{b}^*))}\right)^2 \tag{18}$$

In which

$$H_{coe}(\bar{b}^*) \equiv \frac{(1-\delta)}{\delta}\bar{b}^*[1-\bar{b}^*] + \delta\bar{k} \tag{19}$$

This term H_{coe} does not depend on γ or the loan rates. Intuitively, it captures banks' total management costs. Compared to no CBDC case in which $H_n = \delta\bar{k}$, the bank's total management cost now depends on the equilibrium fraction of debit/CBDC users, some of whom their projects generate zero return at $t = 2$. The proposition shows at the equilibrium, both types of lending are active. The loan rates are endogenously determined and one type of payment cannot completely drive out the other one due to the clearing conditions for loan rates.

To see the determinants of entrepreneurs' choices, we can combine (13) and (14) and see the equilibrium $\bar{b}^*$ satisfies:

$$\bar{b}^* = \frac{\beta + (1-\delta)\bar{k}}{\beta + (\frac{1}{\delta}-1)} \tag{20}$$

Since $\bar{k}$ is small (smaller than 1), we can see $\frac{1}{\delta} - 1 > (1-\delta)\bar{k}$ holds, and the equilibrium $\bar{b}^* < \hat{b} = 1$. A positive fraction of entrepreneurs with $\bar{b}^* \leq b^i \leq 1$ chooses to use the CBDC payment option. The management cost $\bar{k}$ jointly determines this choice with the marginal information loss β. When the cost of using CBDC payment becomes higher, the equilibrium $\bar{b}^*$ will be higher, and a larger fraction of borrowers will stick to using the debit card payment.

Proposition 4. *As marginal information loss β is higher, the equilibrium management cost k^*_{dc} increases, and a larger fraction of borrowers will stay using the debit card payment option.*

With CBDC payments, the equilibrium management cost is higher for anonymous payments because anonymity leads to information loss on the banks' side. When the marginal information loss is higher (bigger β), the equilibrium $\bar{b}^*$ corresponds to higher equilibrium debit card users, and it is costly to manage anonymous payment users' loans. Note that this model is general as it allows the management cost k_{dc} to be endogenously determined.

4.2 Comparison: Stability and Welfare

Now, we can compare the stability and welfare with and without CBDC cases. If we compare the equilibrium condition (18) with no CBDC case's condition (12), the difference lies between the terms H_n and H_{coe}. It can be seen:

$$\Delta H = H_n - H_{coe} = (k^*_{dc} - \delta \bar{k}) \cdot [1 - \bar{b}^*] \tag{21}$$

So $H_n > H_{coe}$, and the difference captures the additional costs the bank takes. It represents the fraction of anonymity borrowers, $1 - \bar{b}^*$, each incurs an additional expected cost $k^*_{dc} - \delta\bar{k}$ on the bank because the bank cannot terminate unprofitable projects. This interpretation is easier to see if we drop the assumption that k_{dc} is increasing in CBDC users. If, again, we simply assume $k_{dc} = \bar{k}$ then $\Delta H = (1-\delta)\bar{k} \cdot [1 - \bar{b}^*]$. Since the bank cannot kick out nonprofitable projects under CBDC payment, these borrowers incur an expected loss of $\bar{k}$ with probability $1 - \delta$. What is more, the corresponding $\bar{b}^* = \delta\bar{k}$, capturing the *expected* cost of using CBDC since the choice of payment option is determined before the return is realized. The entrepreneurs with $b^i > \delta\bar{k}$ will always have their projects matured at $t = 2$, similar to the planner's solution in which there is an optimal threshold $0 < b^* < \bar{k}$.

In this sense, CBDC is a commitment device not to reveal information. The bank is now involved in more risk-sharing with the borrowers compared to no CBDC case, and the lending is more costly. As a result, the bank's investment return decreases at the equilibrium when the less informative means of payment (CBDC) substitutes the traditional informative one (debit card). Importantly, this leads to changes in the bank's portfolio choice that balances investment return and liquidity shortage. The more costly CBDC, however, does not necessarily lead to unstable banks. The following Proposition states this.

Proposition 5. *Compared with no CBDC case, the bank is more stable with $\gamma^* > \gamma^*_n$ and $\bar{\pi} > \bar{\pi}_n$.*

When CBDC is introduced, the loans become more costly, and the CBDC rate is higher than the debit rate, as seen from condition (14). The bank now chooses a more liquid investment portfolio, contributing to a higher γ^*. Thus, the reserve threshold $\bar{\pi}$ is higher in the equilibrium, corresponding to a lower likelihood of experiencing the liquidity shortage at $t = 1$. In this way, the anonymous CBDC indirectly promotes bank stability since it enforces a more liquid portfolio when there is more risk-sharing sourced from introducing the less informative means of payment.

Two factors lower the bank's investment return. As discussed in the previous paragraph, the first and most important factor is the risk-sharing CBDC users bring in. The second is the bank's ability to manage its loans is hampered. This is captured by our assumption in k_{dc} and $k^*_{dc} > \bar{k}$ at the equilibrium, reflecting the point that monitoring the less informative assets is harder. Both factors contribute to a more liquid portfolio because the bank responds to information loss by balancing investment return and chances of liquidity shortage.

The impact of the privacy-preserving payment can also be viewed as an outside option that enables the borrowers to fully capture the privacy gains, which leads to a lower aggregate demand for loans. The following graph compares the equilibrium in the lending market without and with CBDC payment.[6] This downward shift in aggregate demand for loans corresponds to the higher CBDC rate at the equilibrium, which can be seen from condition (13).

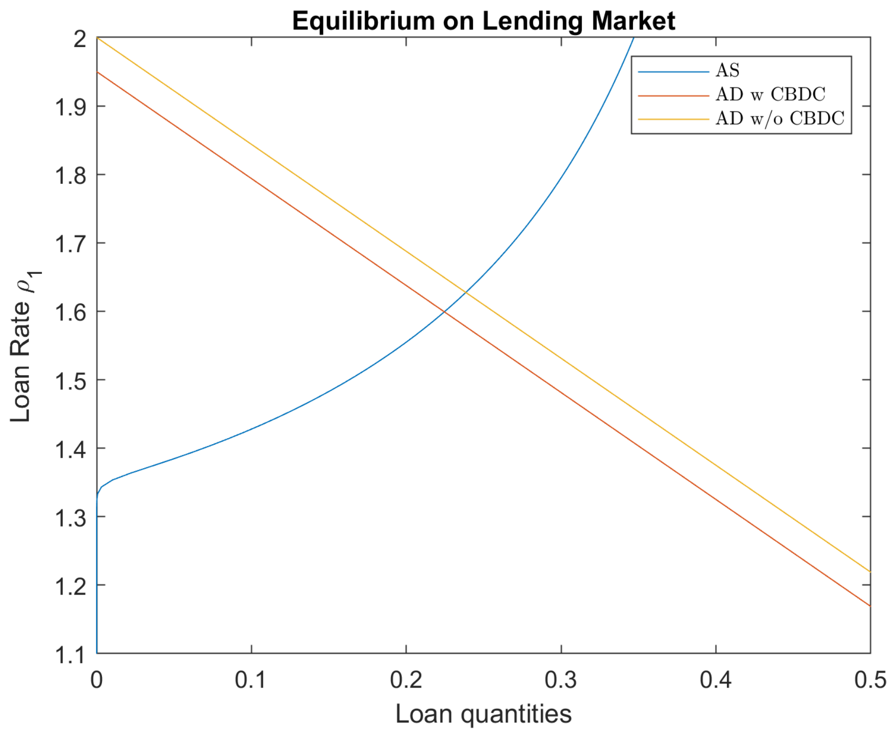

Figure 2. Equilibrium in the Lending Market

The introduction of CBDC payment contributes to higher welfare for entrepreneurs. This can be seen from the equilibrium debit card rate ρ_1^*, which is lower than the no CBDC rate ρ_n^*. The debit card payment users benefit from such a lower loan rate, though they still face the probability of loan termination. This result comes from the fact that at the equilibrium, the aggregate demand for loans decreases due to more risk-sharing with banks. The bank's return is lower from entrepreneurs using CBDC payment, while the market clearing condition enforces the return on both types of lending holds equal. As a result, the entrepreneurs using

[6] $\delta = 0.8$, $R = 2$, $\bar{k} = 0.1$ and β=0.4.

debit card payments benefit from the lower loan rate even though they do not value privacy enough. Meanwhile, the CBDC payment users are also better off because they are willing to accept a higher loan rate ρ_0^* to preserve privacy and shift to the less informative CBDC payment. Thus, both types of entrepreneurs are better off, and the bank depositors are worse off because of lower investment returns from risk sharing.

Proposition 6. *The equilibrium loan rate satisfies* $\rho_1^* < \rho_n^*$. *Compared with no CBDC case, entrepreneurs using both types of payment are better off; and depositors are worse off.*

In sum, introducing CBDC payment favors the entrepreneurs and lowers the welfare of depositors. The equilibrium bank returns are lower due to the risk-sharing and higher management costs. Compared to no CBDC case, introducing anonymity CBDC brings the welfare tradeoff between the lender (depositors) and the borrowers (entrepreneurs). However, the impact on aggregate welfare in the economy is ambiguous and may not necessarily coincide with the social optimal.

In addition, recall that the socially optimal solution requires for some $b^i > b^*$ the project should always be allowed to mature. Introducing CBDC *partially* solves this problem because those entrepreneurs with $b^i > \bar{b}^*$ choose CBDC that can hide information from the bank. However, the social optimal b^* coincides with $\bar{b}^*$ only for specific values. Nevertheless, compared to the no CBDC case, there are welfare gains for all entrepreneurs. In the next section, I show that when the CBDC rate ρ_0 is a policy tool that the central bank can choose, this partial resolution and welfare improvement can also be achieved.

5 CBDC and Information: the Central Bank Perspective

5.1 CBDC Rate as A Choice Variable

It has been widely discussed that the CBDC rate can be a new policy tool for the central bank to achieve its policy goals. In this paper's context, the CBDC rate can balance the tradeoff between borrowers and lenders. For instance, one can think that ρ_0 is influenced by the ability to pay interests by the central bank upon usage. Compared to the debit rate ρ_1, a policy rate ρ_p that replaces ρ_0 provides the central bank an additional policy tool to influence the economy. It affects the bank's portfolio choice and also captures the role of traditional policy rate: By choosing a higher CBDC rate ρ_0, the central bank also pushes up the equilibrium debit rate ρ_1 as in (14). Thus, it discourages borrowings, and netrepreneurs' outputs are lower. On the other hand, when the CBDC rate is sufficiently low, it makes investment less profitable with lower depositor welfare while the bank is more liquid. The central bank's objective function can be written as the following:

$$
\begin{aligned}
\max_{\rho_p} \quad & \int_0^{\bar{\pi}} ln(\gamma_n + (1-\gamma_n)R_b) \cdot f(\pi)d\pi + \int_{\bar{\pi}}^1 [\pi ln(\frac{\gamma_n}{\pi}) + (1-\pi)ln(\frac{(1-\gamma_n)R_b}{1-\pi})]f(\pi)d\pi \\
& + \delta^2(R-\rho_p) + W_b \\
\text{s.t.} \quad & 1-\gamma = \int_{\bar{\pi}}^1 F(\pi)d\pi \\
& \bar{\pi} = \frac{1}{1+(\frac{1}{\gamma}-1)R_b} \\
& R_b = \delta\rho_p - k_{dc}
\end{aligned}
$$

In which W_b is the term[7] capturing privacy gains that does not depend on ρ_p. The bank's return R_b is expressed as a function of ρ_p, which is implicitly determined by the clearing condition (14). We can see the tradeoff between borrowers and lenders.

Note by choosing ρ_p, the welfare level can coincide with the social optimal. And when the borrowers and lenders are *not* equally weighted, ρ_p^* is more important to balance such welfare tradeoff as it will be harder to achieve socially optimal when both ρ_0, ρ_1 are market-based. Note the controlled CBDC rate does not influence the equilibrium CBDC users $\bar{b}^*$ as the debit rate also changes, so the relative fraction of users captured by $\bar{b}$ does not change. Lastly, ρ_p also has a potential role in affecting bank stability. The choice of γ, thus the probability of liquidity shortage, is indirectly influenced by ρ_p.

5.2 Management Cost k_{dc}

Another interesting point is when CBDC is introduced, the central bank naturally acquires the information that flows outside the traditional debit payment. Whether and how the central bank makes use of the information remains unknown.

The anonymous CBDC payment that potentially substitutes the informative payment can lead to more risk-sharing between borrowers and lenders. The information loss may also deteriorate the bank's ability to manage the loans, reflecting from $k_{dc} > \bar{k}$ at the equilibrium. Meanwhile, the CBDC is issued and managed by the central bank, which serves public goals and has no incentive to make profits. The central bank naturally has an advantage in managing the payment information within the digital currency payment system. This brings in the discussion of whether and how the payment information should be managed. The

[7] In particular, $W_b = \int_0^{\bar{b}} \delta b^i + \int_{\bar{b}}^1 b^i + \int_0^{\bar{b}} (1-\delta)\bar{b}$

implication relies on the interaction between the loan rates ρ_0^* and ρ_1^*, which can respond differently to the information loss. Importantly, managing the CBDC payment information to lower the bank's management cost may not necessarily be welfare-improving.

Consider that the central bank reveals all the payment information to the bank. Then, the bank could fully acquire entrepreneurs' payment information and observe investment returns again. This coincides with the no CBDC case in which all investments can be terminated at $t = 1$ if returns are zero. There will be no risk sharing again between the borrowers and banks. In this sense, providing all the information to the bank, i.e., allowing banks to observe the investment return early reveals too much information.

Another potential influence of the central bank providing the information is on the management cost k_{dc}. Recall the CBDC management cost $k_{dc} = \bar{k} + \beta(1 - \bar{b})$, which will be higher if more borrowers hide information from banks. The central bank may play a role in alleviating this effect. For instance, providing information back to banks prevents moral hazard, so the marginal information cost β is lower. Within this paper's context, suppose the central bank can affect the marginal information cost of the bank. This is achieved by allowing the central bank to have a choice variable $\beta_p \in [0, 1]$ to replace the coefficient β:

$$k_{dc} = \bar{k} + \beta_p(1 - \bar{b})$$

By choosing β_p, the central bank can affect the equilibrium k_{dc} and the fraction of CBDC borrowers. As suggested in Proposition 4, the central bank could lower the cost k_{dc} by lowering β_p. If $\beta_p = 0$, the bank bears no additional management costs from CBDC users, i.e., $k^*_{dc} = \bar{k}$, the model is simplified and the CBDC payment becomes only a commitment devise to ensure borrowers receive privacy gains. If $\beta_p = 1$, the central bank facilitates risk-sharing, and borrowers capture privacy gain more easily. However, with both payments coexisting, the changing β_p and k_{dc} may adversely affect the informative payment users. In particular, as β_p decreases, the equilibrium ρ_0 decreases while the debit payment loan rate ρ_1 may decrease or increase. The following figure provides a numerical example showing the pattern of ρ_0^* and ρ_1^* and their relation with the number of informative payment users $\bar{b}^*$ at the equilibrium. [8]

[8] $\delta = 0.8$, $R = 2$, $\bar{k} = 0.1$ and $\beta_p \in [0.05, 0.5]$.

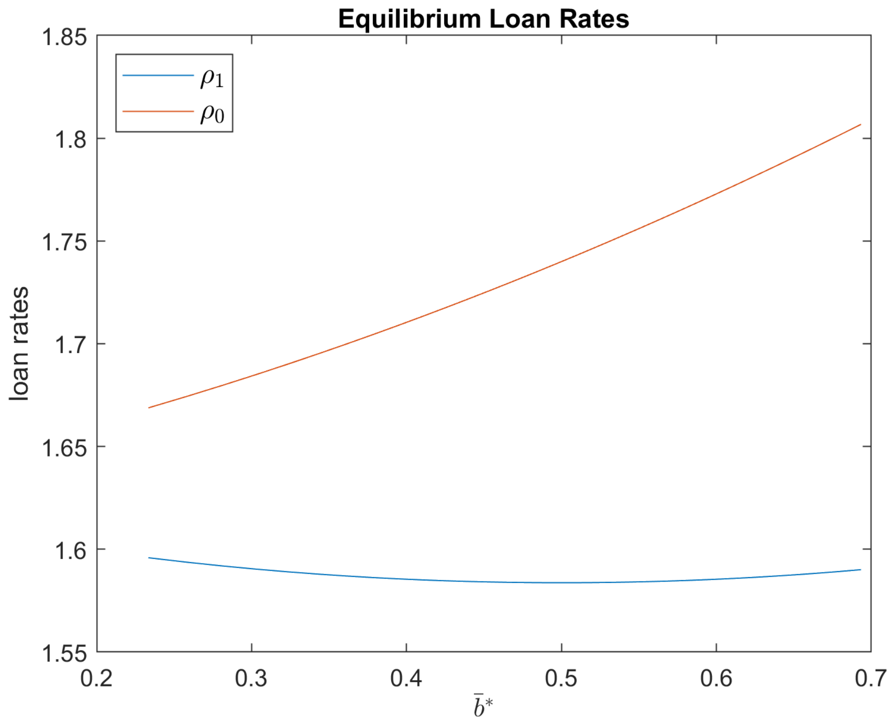

Figure 3. Equilibrium Loan Rates

When β_p decreases, the per loan management cost k_{dc}^* decreases, making the CBDC payment more attractive. The fraction of debit card payment users, $\bar{b}^*$, becomes smaller. It is less costly to lend to a CBDC entrepreneur and the CBDC rate ρ_0^* decreases. Meanwhile, the equilibrium bank return changes and the *relative* attractiveness of CBDC to debit payment may decrease or increase, captured by the difference in aggregate lending cost $\Delta H = (k_{dc}^* - \bar{k}) \cdot (1 - \bar{b}^*)$ as in condition (21). In particular, ΔH has the extensive-intensive tradeoff: when β_p decreases, the per loan cost k_{dc}^* decreases and the amount of CBDC users increases. Thus, the changing β_p has an ambiguous effect on the debit card rate ρ_1^*. Intuitively, if β_p is sufficiently small with a low k_{dc}^*, then the aggregate management cost $H_{\bar{b}^*}$ also decreases even though many entrepreneurs choose CBDC payment. The debit card payment rate ρ_1^* will thus increase; the fund supply for debit card payment users decreases relative to the supply for CBDC users when the market clears. On the other hand, if β_p is relatively high, a decrease in β_p lowers the per loan k_{dc}. But with more CBDC users, the aggregate cost ΔH increases. This contributes to lower equilibrium return, and the ρ_1^* is lower when the market clears.

This result suggests an externality in managing the CBDC payment. When there is no CBDC, a decrease in the bank's management cost $\bar{k}$ contributes to higher bank returns and lower loan rates. The gains from lowering costs always benefit the bank depositors and the entrepreneurs. With CBDC payment, a decrease in CBDC cost k_{dc} may lead to a higher loan rate ρ_1 that could hurt the informative payment users. Entrepreneurs using informative payment can be adversely affected by the rising loan rate. The central bank may need to balance the welfare between the lender and borrowers and between the two types of payment users.

6 Conclusion

In this paper, I study the potential macroeconomic implications of an anonymous CBDC on bank stability and welfare in the lending market. The anonymous payment method can preserve entrepreneurs' privacy while substituting for the traditional, informative payment method. The information available for the bank is relatively limited compared to the case that the bank can better monitor and manage the loans when the entrepreneurs can only have an exclusive arrangement with the bank. The anonymous CBDC payment serves as a commitment device and allows borrowers to fully capture their non-pledgeable gains. It enforces more risk sharing between the bank and entrepreneurs, leading to lower bank returns and less demand for lending with a higher anonymous loan rate. These results hold even when the information loss does not hamper banks' ability to monitor the loans. The bank that balances liquidity shortage and loan return will optimally respond by choosing a more liquid portfolio, eventually contributing to a more stable bank. This result is contrary to the concern that the loss of information would increase banks' risks and make them unstable.

Introducing anonymous payment favors the entrepreneurs while decreasing the bank depositors' welfare. Interestingly, entrepreneurs who value their privacy less can also benefit from introducing CBDC while continuing to use the informative payment. This is because the loan rate of the informative CBDC payment becomes lower as the aggregate demand decreases. The anonymous and informative loan rates are inter-connected, so the central bank can use the CBDC to influence the bank's portfolio choice and management costs. In this sense, the central bank should also be cautious in managing the payment information that flows within the CBDC payment system.

References

Aghion, Philippe and Patrick Bolton (1992) "An incomplete contracts approach to financial contracting," *The review of economic Studies*, 59 (3), 473–494.

Andolfatto, David, Aleksander Berentsen, and Christopher Waller (2014) "Optimal disclosure policy and undue diligence," *Journal of Economic Theory*, 149, 128–152.

Berg, Tobias, Valentin Burg, Ana Gombović, and Manju Puri (2020) "On the rise of fintechs: Credit scoring using digital footprints," *The Review of Financial Studies*, 33 (7), 2845–2897.

Bindseil, Ulrich (2020) "Tiered CBDC and the Financial System," *ECB Working Paper*, https://ssrn.com/abstract=3513422.

Champ, Bruce, Bruce D Smith, and Stephen D Williamson (1996) "Currency elasticity and banking panics: Theory and evidence," *Canadian Journal of Economics*, 828–864.

Chiu, Jonathan, Seyed Mohammadreza Davoodalhosseini, Janet Jiang, and Yu Zhu (2023) "Bank market power and central bank digital currency: Theory and quantitative assessment," *Journal of Political Economy*, 131 (5), 1213–1248.

Diamond, Douglas W and Philip H Dybvig (1983) "Bank runs, deposit insurance, and liquidity," *Journal of political economy*, 91 (3), 401–419.

European Central Bank (2020) "Report on a digital euro," October, www.ecb.europa.eu/euro/html/digitaleuro-report.en.html.

Fernández-Villaverde, Jesús, Daniel Sanches, Linda Schilling, and Harald Uhlig (2021) "Central bank digital currency: Central banking for all?" *Review of Economic Dynamics*, 41, 225–242, https://doi.org/10.1016/j.red.2020.12.004, Special Issue in Memory of Alejandro Justiniano.

Garratt, Rodney J and Maarten RC Van Oordt (2021) "Privacy as a public good: a case for electronic cash," *Journal of Political Economy*, 129 (7).

Gross, Jonas and Jonathan Schiller (2021) "A Model for Central Bank Digital Currencies: Implications for Bank Funding and Monetary Policy," https://ssrn.com/abstract=3721965.

Hart, Oliver and John Moore (1998) "Default and renegotiation: A dynamic model of debt," *The Quarterly Journal of Economics*, 113 (1), 1–41.

He, Zhiguo, Jing Huang, and Jidong Zhou (2023) "Open banking: Credit market competition when borrowers own the data," *Journal of Financial Economics*, 147 (2), 449–474.

Hirshleifer, Jack (1971) "The private and social value of information and the reward to inventive activity," in *Uncertainty in economics*, 541–556: Elsevier.

Izumi, Ryuichiro (2021) "Opacity: Insurance and fragility," *Review of Economic Dynamics*, 40, 146–169.

Kang, Kee-Youn (2021) "Digital Currency and Privacy," *Working Paper*, `https://ssrn.com/abstract=3838718`.

Keister, Todd and Cyril Monnet (2020) "Central bank digital currency: Stability and information," *Working paper, Rutgers University and University of Bern*.

Keister, Todd and Daniel Sanches (2023) "Should central banks issue digital currency?" *The Review of Economic Studies*, 90 (1), 404–431.

Kim, Young Sik and Ohik Kwon (2023) "Central bank digital currency, credit supply, and financial stability," *Journal of Money, Credit and Banking*, 55 (1), 297–321.

Lee, Michael and Rodney Garratt (2021) "Monetizing Privacy," *FRB of New York Staff Report* (958).

Monnet, Cyril., Asgerdur. Petursdottir, and Mariana Rojas-Breu (2021) "Central Bank Account for All: Efficiency and Risk Taking," *Working Paper*.

Parlour, Christine A, Uday Rajan, and Haoxiang Zhu (2022) "When fintech competes for payment flows," *The Review of Financial Studies*, 35 (11), 4985–5024.

Piazzesi, Monika and Martin Schneider (2020) "Credit lines, bank deposits or CBDC? competition and efficiency in modern payment systems," *Unpublished, Stanford University*.

Williamson, Stephen D. (2021) "Central bank digital currency and flight to safety," *Journal of Economic Dynamics and Control*, 104146, https://doi.org/10.1016/j.jedc.2021.104146.

Appendix: Proofs

I begin by showing a threshold $\bar{\pi}$, above which the optimal choice of $\alpha(\pi) = 1$ so the bank runs out of liquidity.

In the bank's problem, after the random π is realized (call it ex-post), the bank makes the state-contingent offer $\{c_1(\pi), c_2(\pi), \alpha(\pi)\}$ based on the realized π. Denote the bank's lending return as R_b. For now, we simply take R_b as given and ignore $\mathbb{I}^i$. For a given γ_n, the ex-post problem can be expressed as:

$$
\begin{aligned}
\max_{c_1(\pi), c_2(\pi), \alpha(\pi)} \quad & \pi \cdot ln(c_1(\pi)) + (1-\pi) \cdot ln(c_2(\pi)) \\
\text{s.t.} \quad & \pi c_1(\pi) = \alpha(\pi)\gamma_n \\
& (1-\pi)c_2(\pi) \leq \gamma_n - \alpha(\pi)\gamma_n + (1-\gamma_n)R_b \\
& 0 \leq \alpha(\pi) \\
& \alpha(\pi) \leq 1
\end{aligned}
$$

Denote the constraints' corresponding multipliers as λ_1 to λ_4. Take the first orders:

$$
\begin{aligned}
\frac{\pi}{c_1(\pi)} - \pi\lambda_1 = 0 \\
\frac{1-\pi}{c_2(\pi)} - \pi\lambda_2 = 0 \\
\lambda_1\gamma_n - \lambda_2\gamma_n + \lambda_3 - \lambda_4 = 0
\end{aligned}
$$

Note $\alpha(\pi) \neq 0$ otherwise $c_1(\pi) = 0$, which cannot be the solution. If $0 < \alpha(\pi) < 1$, then $\lambda_3 = \lambda_4 = 0$ and $\lambda_1 = \lambda_2 = \frac{1}{c_1(\pi)} = \frac{1}{c_2(\pi)}$. From the constraints, we have:

$$
\begin{aligned}
c_1(\pi) = c_2(\pi) = \gamma_n + (1-\gamma_n)R_b \\
\pi[\gamma_n + (1-\gamma_n)R_b] = \alpha(\pi)\gamma_n
\end{aligned}
$$

Recall that the second condition should also satisfy $\alpha(\pi) \leq 1$, which depends on γ_n and π. The threshold $\bar{\pi}$ is defined as $\alpha = 1 = \frac{\bar{\pi}[\gamma_n + (1-\gamma_n)]R_b}{\gamma_n}$, or equivalently $\bar{\pi} = \frac{\gamma_n}{\gamma_n + (1-\gamma_n)R_b}$. For those realized $\pi > \bar{\pi}$, $\alpha(\pi) = 1$. And $\alpha(\pi) = 1$ is the third scenario with:

$$
\begin{aligned}
\pi c_1(\pi) = \gamma_n \\
(1-\pi)c_2(\pi) = (1-\gamma_n)R_b
\end{aligned}
$$

In sum, once π is realized, there is a threshold $\bar{\pi} = \frac{\gamma_n}{\gamma_n + (1-\gamma_n)R_b}$ that if $\pi \leq \bar{\pi}$, $c_1(\pi) = c_2(\pi) = \gamma_n + (1-\gamma_n)R_b$; if $\pi > \bar{\pi}$, then $c_1(\pi) = \frac{\gamma_n}{\pi}$, $c_2(\pi) = \frac{(1-\gamma_n)R_b}{1-\pi}$ and the bank runs out of liquidity.

These ex-post solutions are conditioned on π and γ_n. Now we look at the bank's ex-ante choice of γ_n. The bank's optimization problem can be written as:

$$\begin{aligned}
\max_{\gamma_n} \quad & \int_0^{\bar{\pi}} ln(\gamma_n + (1-\gamma_n)R_b) \cdot f(\pi)d\pi + \int_{\bar{\pi}}^1 [\pi ln(\frac{\gamma_n}{\pi}) + (1-\pi)ln(\frac{(1-\gamma_n)R_b}{1-\pi})]f(\pi)d\pi \\
\text{s.t.} \quad & \bar{\pi} = \frac{\gamma_n}{\gamma_n + (1-\gamma_n)R_b} \\
& 0 \leq \gamma_n \\
& \gamma_n \leq 1
\end{aligned}$$

I denote the value of depositors' expected utilities in the maximization problem as $V(\gamma, R_b)$. These discussions above help to show the proofs of Propositions.

Proposition 1. *There exists a privacy gain level $b^* > 0$ such that for $b^i > b^*$, the entrepreneur's project should always be funded to mature in period 2. When $k_{dc} = \bar{k}$, $b^* < \bar{k}$.*

Proof. We can use $k_{dc} = \bar{k}$ to illustrate. Suppose CBDC payment option does not incur any additional management cost so $k_{dc} = \bar{k}$ is a constant. The planner chooses $\gamma, \rho, \mathbb{I}^i$ to maximize social welfare. We can express the depositors' welfare as $V(\gamma, R_b)$, so:

$$\begin{aligned}
\max_{\rho^*, \gamma^*, \mathbb{I}^i} \quad & \int_0^1 [\delta^2(R-\rho^*) + \mathbb{I}^i \cdot b^i]db^i + V(\gamma, R_b) \\
\text{s.t.} \quad & \bar{\pi} = \frac{1}{1 + (\frac{1}{\gamma} - 1)R_b} \\
& R_b = \delta\rho - \bar{k} \cdot \mathbb{I}^i
\end{aligned}$$

For certain decision $\mathbb{I}^i$, we can see the marginal gain by allowing project i to mature (after knowing its future return) is b^i.

The cost of allowing an unprofitable project to mature comes solely from R_b. So, the marginal cost can be expressed as:

$$\begin{aligned}
\frac{\partial V(\mathbb{I}^i)}{\partial \mathbb{I}^i} &= \frac{\partial V(\mathbb{I}^i)}{\partial R_b} \cdot \frac{\partial R_b}{\partial \mathbb{I}^i} \\
&= (\int_0^{\bar{\pi}} \frac{1-\gamma^*}{\gamma^* + (1-\gamma^*)R_b} d\pi + \int_{\bar{\pi}}^1 \frac{1-\pi}{R_b} d\pi) \cdot \bar{k} \\
&< (\int_0^{\bar{\pi}} \frac{1}{R_b} d\pi + \int_{\bar{\pi}}^1 \frac{1}{R_b} d\pi) \cdot \bar{k} \\
&= \frac{1}{R_b} \cdot \bar{k} \\
&< \bar{k}
\end{aligned}$$

So the marginal project with $\mathbb{I}^i = 1$, i.e., the entrepreneur has privacy gain b^* and is allowed to continue the project satisfies:

$$(1-\delta)b^* - \frac{\partial V(\mathbb{I}^i)}{\partial \mathbb{I}^i} = 0$$

should also satisfy $b^* < \bar{k}$. In other words, for those $b^i \geq b^* = \bar{k}$, the marginal gain is larger than the marginal costs and should be allowed to mature even if the project is unprofitable for the bank.

Note when k_{dc} can be larger than $\bar{k}$, the same mechanism works. The difference is that the threshold b^* is larger because the bank's cost will be higher. Still, a smaller fraction of entrepreneurs should always be allowed to have their projects mature. □

Now I jointly show **Lemma 1**, **Propositions 2**, and **Proposition 3** below.

Proof. I start by showing the bank will always terminate unprofitable loans. When the bank observes the loan generates a zero return, not terminating it contributes to a lower return $R' < R$ due to the management cost. Let the optimal choice of cash reserve as γ^*, we can use the envelope theorem to get:

$$\frac{dV(\gamma^*(R_b), R_b)}{dR_b} = \frac{\partial V(R_b)}{\partial R_b} = \int_0^{\bar{\pi}} \frac{1-\gamma^*}{\gamma^* + (1-\gamma^*)R_b} d\pi + \int_{\bar{\pi}}^1 \frac{1-\pi}{R_b} d\pi > 0$$

In which $\bar{\pi} = \frac{\gamma^*}{\gamma^*+(1-\gamma^*)R_b}$. So the bank will always terminate unprofitable loans to achieve higher expected utilities for depositors. This result delivers Lemma 1.

Then, I turn to find the optimal γ^*. First, notice $\gamma_n \neq 0$ otherwise $c_1(\pi) = 0$, which cannot be the solution. Second, if $0 < \gamma_n < 1$, we can get the first order condition by applying the Leibniz integral rule:

$$\begin{aligned}\frac{dV(\gamma)}{d\gamma} = &\frac{R_b}{(\gamma + (1-\gamma)R_b)^2} \cdot ln(\gamma + (1-\gamma)R_b) \cdot f(\bar{\pi}) + \int_0^{\bar{\pi}} \frac{1-R_b}{\gamma + (1-\gamma)R_b} f(\pi) d\pi \\ &- \frac{R_b}{(\gamma + (1-\gamma)R_b)^2} \cdot [\bar{\pi} ln(\frac{\gamma}{\bar{\pi}}) + (1-\bar{\pi}) ln \frac{(1-\gamma)R_b}{1-\bar{\pi}}] \cdot f(\bar{\pi}) + \int_{\bar{\pi}}^1 [\frac{\pi}{\gamma} - \frac{1-\pi}{1-\gamma}] f(\pi) d\pi\end{aligned}$$

The first line is the derivative of the first integral and the second line comes from the second integral of the objective function. In addition, the first term in the first line cancels out the first part of the second line with $\bar{\pi} = \frac{\gamma}{\gamma+(1-\gamma)R_b}$. So we can further simplify the equation:

$$\begin{aligned}\frac{dV(\gamma)}{d\gamma} &= \int_0^{\bar{\pi}} \frac{1-R_b}{\gamma + (1-\gamma)R_b} f(\pi) d\pi + \int_{\bar{\pi}}^1 \frac{\pi - \gamma}{\gamma(1-\gamma)} f(\pi) d\pi \\ &= \int_0^{\bar{\pi}} \frac{1-R}{\gamma + (1-\gamma)R_b} f(\pi) d\pi + \frac{\pi - \gamma}{\gamma(1-\gamma)} F(\pi)|_{\bar{\pi}}^1 - \int_{\bar{\pi}}^1 \frac{1}{\gamma(1-\gamma)} F(\pi) d\pi \\ &= \int_0^{\bar{\pi}} \frac{1-R_b}{\gamma + (1-\gamma)R_b} f(\pi) d\pi + \frac{1}{\gamma} - \frac{\frac{\gamma}{\gamma+(1-\gamma)R_b} - \gamma}{\gamma(1-\gamma)} F(\bar{\pi}) - \int_{\bar{\pi}}^1 \frac{1}{\gamma(1-\gamma)} F(\pi) d\pi \\ &= \frac{1}{\gamma} - \int_{\bar{\pi}}^1 \frac{1}{\gamma(1-\gamma)} F(\pi) d\pi\end{aligned}$$

In the second line, I apply integral by parts for the second term of the equation. In the third line, the first and the last terms cancel out. Imposing $\frac{dV(\gamma)}{d\gamma} = 0$, we get equation (8) as in the main content:

$$1-\gamma=\int_{\bar{\pi}}^{1}F(\pi)d\pi \tag{22}$$

It solves for the bank's choice of γ for a given investment return R_b that the representative bank takes as given.

We use (22) and the market clearing condition to find the equilibrium solution of $\{\gamma^*,\rho^*\}$. Imposing $F(\pi)=\pi$ and replacing R_b using the clearing condition (replace R_b with project's return R), we can find the equation that pins down the equilibrium γ^* as:

$$1-\gamma=\frac{1}{2}-\frac{1}{2}\{\frac{1}{1+(\frac{1}{\gamma}-1)[C-\frac{1}{\delta}(1-\gamma)]}\}^2 \tag{23}$$

In which $C=\delta R-\delta k$ for simplifying expression. Denote the left-hand-side as $g_0(\gamma)$ and the right-hand-side as $g_1(\gamma)$. I can show $g_1(\gamma)$ is decreasing and concave when $\gamma\in[\frac{1}{2},1]$. Through some calculations, we can find the first and second order of $g_1(\gamma)$ are:

$$\begin{aligned}\frac{dg_1(\gamma)}{d\gamma}&=-\frac{1}{\gamma^2}(C-\frac{1}{\delta}+\frac{1}{\delta}\gamma^2)\cdot\{\frac{1}{1+(\frac{1}{\gamma}-1)[C-\frac{1}{\delta}(1-\gamma)]}\}^3\\ \frac{d^2g_1(\gamma)}{d^2\gamma}&\equiv[-3+2(1-\gamma)]\cdot(C-\frac{1}{\delta})^2-\frac{3}{\delta^2}\gamma^4-\frac{2\gamma}{\delta}[4\gamma-\delta-1]\cdot(C-\frac{1}{\delta})\end{aligned}$$

When $C>\frac{1}{\delta}$, $g_1(\gamma)$ decreases in γ. Intuitively, this condition requires the expected return on bank's investment (entrepreneurs' project) should be larger than one. In addition, note the optimal $\gamma^*\in(\frac{1}{2},1]$ so the last term of the second order function is non-negative whenever $\delta\in[0,1]$. Thus, $\frac{d^2g_1(\gamma)}{d^2\gamma}<0$ holds and $g_1(\gamma)$ is decreasing and concave in $\gamma\in[\frac{1}{2},1]$.

Now we can see $g_1(\gamma\to 0)=\frac{1}{2}$ and $g_1(1)=0$. As $\gamma\to 1$, the slope $\frac{dg_1(\gamma)}{d\gamma}\to -C$ and will be smaller than -1 whenever $C>1$, which is ensured by $C>\frac{1}{\delta}$. Thus, a single crossing interior point γ^* solves equation (23). The figure below shows this result.

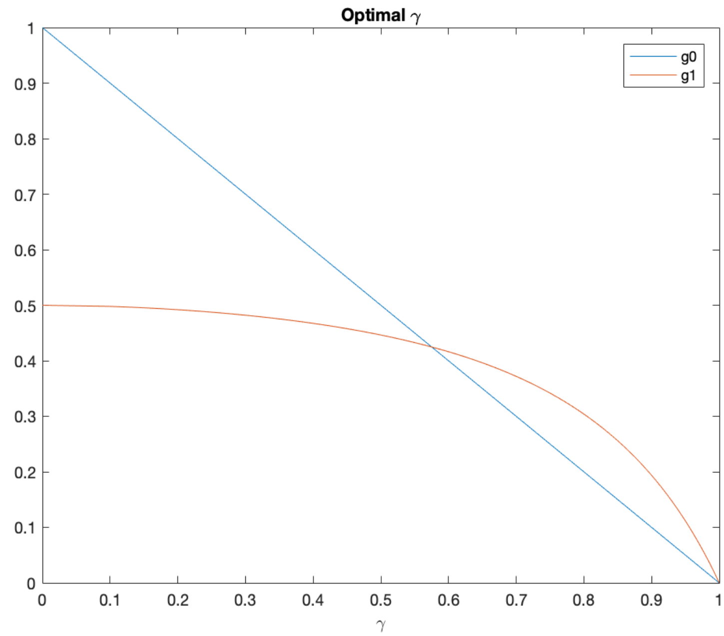

Figure 2. Optimal γ_n

The properties of $g_1(\gamma)$ also suggest that the corner choice $\gamma = 1$ cannot be the solution to the maximization problem. For those $\gamma \in (\gamma^*, 1)$, we have equation (23)'s the left-hand-side smaller than the right-hand-side or equivalently, $\frac{dV(\gamma)}{d\gamma} < 0$ on equilibrium as captured by 22. When $\gamma = 1$, the bank does not lend and $\bar{\pi} = 1$. However, no lending generates lower welfare than interior γ^* because $\frac{dV(\gamma)}{d\gamma} < 0$ holds.

Lastly, when R increases the curve $g_1(\gamma)$ shifts upward. So the solution γ^* decreases. From the equation (8), the threshold $\bar{\pi}$ should also increase for the equality to hold.

□

Proposition 4. *As marginal information loss β is higher, a larger fraction of borrowers will stay using debit card payment. The equilibrium management cost k^*_{dc} increases.*

Proof. The equilibrium fraction of debit card payment user $\bar{b}^*$ can be expressed as:

$$\bar{b}^* = \frac{\beta + (\frac{1}{\delta} - 1)\delta\bar{k}}{\beta + (\frac{1}{\delta} - 1)}$$

$$= 1 - \frac{(\frac{1}{\delta} - 1)(1 - \delta\bar{k})}{\beta + (\frac{1}{\delta} - 1)}$$

It can be shown that $\frac{d\bar{b}^*}{d\beta} > 0$. In addition, the equilibrium CBDC management cost $k^*_{dc} =$

$\bar{k} + \beta(1 - \bar{b}^*)$. Take the derivative to β:

$$\begin{aligned}
\frac{dk^*_{dc}}{d\beta} &= (1 - \bar{b}^*) - \beta\frac{d\bar{b}^*}{d\beta} \\
&= \frac{(\frac{1}{\delta} - 1)(1 - \delta\bar{k})}{\beta + (\frac{1}{\delta} - 1)} - \frac{\beta}{\beta + (\frac{1}{\delta} - 1)} \cdot \frac{(\frac{1}{\delta} - 1)(1 - \delta\bar{k})}{\beta + (\frac{1}{\delta} - 1)} \\
&= \frac{(\frac{1}{\delta} - 1)^2(1 - \delta)\bar{k}}{\beta + (\frac{1}{\delta} - 1)} \\
&> 0
\end{aligned}$$

So that as β increases, both $\bar{b}^*$ and k^*_{dc} increase at the equilibrium. □

Lastly, we can compare the stability result with and without CBDC. Following the same procedure in the proofs of Proposition 2, comparing the liquidity holdings γ in the same figure is easy. The curve $g2$ is the right-hand-side of condition (18).

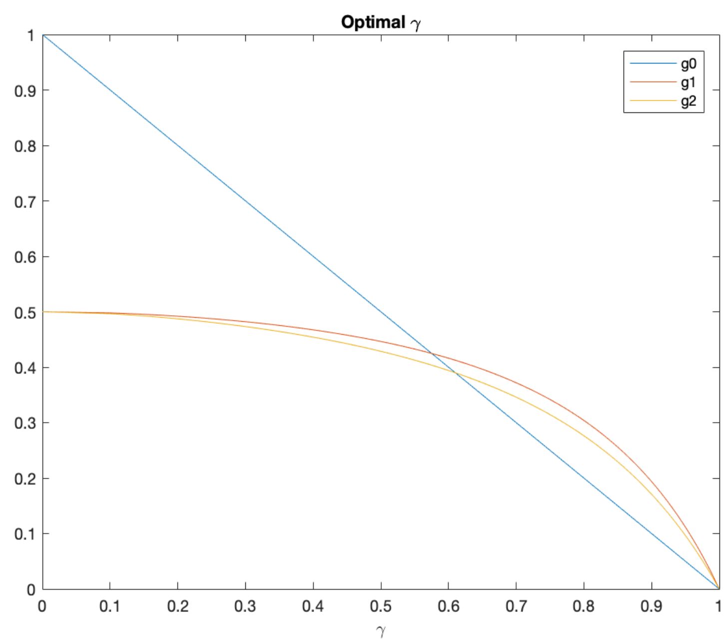

Figure 2. Optimal γ_n

It is easy to see the bank chooses a lower γ due to risk-sharing with the CBDC entrepreneurs. This provides the result of Proposition 5.

We can further draw the demand and supply curves. With CBDC, the aggregate demand is:

$$
\begin{aligned}
AD &= \int_0^{\bar{b}} \delta^2(R-\rho_1) + \int_{\bar{b}}^1 \delta^2(R-\rho_0) \\
&= \delta^2 R - [\int_0^{\bar{b}} (\delta^2\rho_0 - (1-\delta)\bar{b}) + \int_{\bar{b}}^1 \delta^2(R-\rho_0)] \\
&= \delta^2(R-\rho_0) + (1-\delta)\bar{b}^2 \\
&= \delta^2(R-\rho_1) - (1-\delta)(1-\bar{b})\bar{b}
\end{aligned}
$$

The second line uses the clearing condition (14) to replace ρ_0. The last line is the expression in ρ_1 to compare the debit rates with and without CBDC later.

The FOC of γ gives the aggregate supply of funds, which equals to $1-\gamma$, and the condition is:

$$
1-\gamma = \int_{\frac{1}{1+(\frac{1}{\gamma}-1)R_b}}^1 F(\pi)d\pi \tag{24}
$$

In which $R_b = \theta R_{b1} + (1-\theta)R_{b0} = R_{b1} = \delta\rho_1 - \delta\bar{k} = R_{b0} = \delta\rho_0 - k_{dc}$. This holds as the return is the same from either type of lending. Notice that the equilibrium θ can be backed out from condition (17), although it is canceled out in the expressions.