Competition, Bailouts and Financial Fragility*

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Abstract

I study the relationship between competition in the banking sector and financial fragility. I identify a new channel through which increased competition amplifies fragility in the banking system. In equilibrium, a more competitive banking system will be larger because more total liabilities are issued to depositors. In the event of a run, a government with limited resources will choose smaller bailouts as a fraction of banks' liabilities. Depositors anticipate these smaller bailouts, which gives them stronger incentives to run on their banks. In other words, increased competition can make the banking system "too big to save." I show that a form of interest rate ceiling is a more efficient way to promote financial stability in this setting than restricting competition.

JEL codes: G21, G28, E61

Keywords: Bank runs, Competition, Fragility, Bailout, Limited commitment

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1 Introduction

There is a long-running debate about how competition in the banking system affects financial fragility. An extensive literature has studied this question and identified many channels through which the degree of competition affects incentives in a way that makes the banking system either more or less stable[1]. For instance, more competition tends to lower bank profits, which may lead banks to take more risk and thereby increase fragility. At the same time, competition tends to decrease loan rates, which may lead borrowers from banks to take less risk and thereby increase financial stability. Understanding the various ways in which competition affects stability is important for the design of the public policy. Following the global financial crisis of 2008-9, many countries have adopted regulatory reforms that significantly alter the competitive environment. In addition, crisis-era bank mergers sharply increased concentration in the banking sector in the U.S. Designing effective competition policy requires understanding how these and other changes affect the likelihood of a future crisis.

In this paper, I revisit the questions of whether and how the level of competition in the banking sector affects financial fragility. I identify a new channel through which increased competition amplifies fragility. This channel is created by government bailouts, which have been an important feature in recent financial crises. In my model, increased competition drives banks to offer higher payoffs to attract depositors. These higher payoffs imply that, in equilibrium, the banking system is larger in the sense of having issued more liabilities to depositors. In the event of a crisis, the government will find it optimal to intervene and bail out depositors facing losses. When the banking system is larger, the government's bailout will cover a smaller fraction of depositors' losses. Anticipating this fact, depositors will have a stronger incentive to run on their banks. In other words, increased competition tends to undermine the government's ability to stabilize the banking system.

This bailout amplification channel is consistent with the "too big to save" problem discussed by Kunt and Huizinga (2013) and Barth and Wihlborg (2016). While there was an intense discussion of "too big to fail" after the global financial crisis, the fiscal and political costs associated with bank bailouts and events in the European debt crisis suggested there might also be a separate "too big to save" problem in the banking system. Iceland is a prime example. Iceland's banking sector grew rapidly in the early 2000s as the three largest banks competed to attract deposits from domestic and abroad. The banking system's liabilities rose to about nine times GDP in 2007, comparing to the United Kingdom's peak of 5.5 during the same period (Kunt and Huizinga, 2013). In the collapse of 2008, the deposit guarantee fund was insufficient to meet depositors' claims as all three major banks were nearing failure. Given this, individual depositors had the incentive to withdraw their funds before the banks failed or were taken over. In other words, the fact that the banking system was too big to save gave depositors an incentive to run on their banks, which exacerbated

[1] See, for example, the reviews by Carletti (2007), Vives (2011, 2016).

the crisis.

I construct a model that combines elements from the literatures on bank runs and competition. I use a modern version of Diamond and Dybvig (1983) bank run model with limited commitment, as in Ennis and Keister (2010). I introduce a government that can provide bailouts as in Keister (2016). If a bank run occurs, the government with limited resources will choose bailout payments to banks as a best response to the situation at hand. As in the previous literature, providing bailouts may or may not be sufficient to remove depositors' incentive to run on their banks. I measure financial fragility by the set of parameter values for which a bank run equilibrium exists. My interest is in how this measure of fragility is affected by the degree of competition. To introduce competition into this framework, I follow Salop (1979) in modelling spatial competition on a circular market. In this framework, an individual bank maximizes its profit and competes to attract depositors. Competition amplifies fragility through bailout policy: more competition contributes to better-treated depositors and larger total bank liabilities. In the event of a crisis, the limited public resources are then spread more thinly and the equilibrium bailout amount per depositor *decreases*. The anticipation of these smaller bailouts, in turn, gives depositors a higher incentive to run, and the banking system becomes more fragile.

It is useful to focus on the bank's capital structure to see how this mechanism differs from the existing literature. The traditional theoretical link between competition and fragility relies on the role of bank profit as a 'buffer' (Chang and Velasco 2001, Martinez-Miera 2010, Matsuoka 2013 et al.). In those settings, banks pay out profits at the end of the model, once it is known that a crisis will not occur. An increasing competition will decrease bank profit, thus mechanically bringing about a decrease in equity and making banks more likely to be insolvent. In my model, a bank instead must use its profits to cover the entry cost, before knowing whether a crisis will occur. Because these profits no longer serves as a buffer, the traditional channel dissolves, but the bailout channel I identified remains operative. In the absence of bailouts, changes in competition would have no effect on fragility in my model. In this way, I highlight how the importance of the usual competition-fragility relationship is conditional on banks' capital structure. This comparison is consistent with the view described by Vives (2016) that competition does not necessarily make banks more fragile. In the absence of distortions, more competition can lead to lower profits and higher depositor payments without deteriorating financial fragility.

Given that competition does increase fragility through the bailout mechanism described above, I ask whether restricting competition is an efficient way to reduce financial fragility. The answer is no. I show a policy tool similar to an interest rate ceiling can lower financial fragility while delivering a higher level of depositor welfare than restricting competition. An interest rate ceiling policy effectively lowers depositor's incentive to run by forcing banks to offer lower payments to depositors who withdraw early. Under this policy, banks still compete for depositors by offering attractive long term investment payments. which provides a higher welfare relative to a policy of restricting competition. By comparison, restricting competition is costly especially because it allows the fewer banks get higher revenues at the equilibrium.

Related Literature

An extensive literature has studied the effects of banking competition on financial fragility with mixed results. Many of these papers focused on competition's impact on the asset side of banks' balance sheets. For example, Keeyley (1990) points out that competition deteriorates stability by eroding the bank's profit margins and leading to an agency problem. Bankers will take more risk as increased competition lowers future profits. Allen and Gale (2004) show similar results about how increased competition can worsen banks' incentive to take risks. Boyd and De Nicolo (2005) show that a more concentrated banking system is riskier, as the borrowers face a higher loan rate and have higher bankruptcy risk. Borrowers' moral hazard problem further reinforces the risk. Martnez and Repullo (2010) show that banking competition has two competing effects on bank stability. They combine the risk-shifting effect above with a marginal effect that the bank's profit shrinks, showing that its failure rate is a U-shaped function of the degree of competition and there is a competition level that the bank's failure rate is lowest.

Fewer papers discuss possible mechanisms that relate competition to fragility through the liability side of banks' balance sheets. The classic Diamond and Dybvig (1983) framework focus on the liabilities side, but typically assumes perfect competition and remains silent on bank size. Exceptions include Goldstein and Pauzner (2005) and Vives (2014), both applying global-games techniques to modified Diamond and Dybvig settings. In these papers, a higher deposit interest rate suggests higher strategic complementary among depositors. Competition contributes to a higher deposit rate, which increases a depositor's sensitivity to others' actions to run. Thus the increased competition raises the probability of a bank run. Chang and Velasco (2001) study an open-economy version of the Diamond-Dybvig model. They show that a financial liberalization, which changes a monopolistic banking system to a competitive one, increases depositors' welfare, and raises fragility. These results arise because a monopolist provides depositors lower payoffs and therefore is less vulnerable to runs by its depositors. Matsuoka (2013) also provides a comparison between a monopolistic and fully competitive banking system, showing that the parameter regime that holds bank run equilibrium is smaller in the former because of lower depositor payoffs. In both Chang and Velasco (2001) and Matsuoka (2013), banks implicitly use profits as a buffer which ends up being paid out to depositors in the event of bank run.

The closest paper to mine is Gao and Reed (2020), which studies the effect of restricting bailouts on fragility under a discretionary policy regime. They find a limited bailout size can reduce fragility when the exogenous probability of run is high, as the bank recognizes the illiquidity risk and partially internalizes it. They compare a monopolistic to a fully competitive banking system and show that the level of fragility depends on whether the bailout constraint binds. In particular, when the bailout constraint does not bind, a monopolistic bank is less fragile because it offers lower payments and thus has more resources available when a bank run begins. When the bailout constraint binds and the bank run probability is sufficiently high, competitive banks become more prudent but the monopolistic bank does not. As a result, a monopolistic bank can be more fragile.

My model differs from Gao and Reed (2020) in three critical ways. First, banking competition drives the changes in the relative size between banking sector and government bailout. Bankers compete to offer payoffs to attract depositors. The offers rise as competition degree increases, so their liabilities grow relative to the government's fiscal capacity. Second, I construct a more general model that naturally includes those two systems as extreme cases instead of comparing the distinct monopolistic and fully competitive cases. Third, a banker in my model does not hold her earnings in the bank as a buffer, which implies that the usual channel through which higher profit leads to less fragility is absent. I instead explore the new channel that operates through bailouts and the size of the banking system affected by competition. Within my framework, I show that increased competition, together with limiting bailouts, amplifies fragility.

The rest of the paper organizes as follows. Section 2 presents the environment and defines the measure of financial fragility. Section 3 demonstrates how competition amplifies fragility through the bailouts channel, and how the model differs from existing approaches. Section 4 discusses different policies to limit financial fragility, showing that restricting competition is not an optimal tool. Section 5 concludes.

2 The Model

In this section, I describe the environment and define the measure of financial fragility I use throughout the analysis.

2.1 Environment

There are three periods, $t = 0, 1, 2$. There is a continuum of depositors with measure 1 distributed evenly on a circular market with length 1, as shown in Figure 1. Each depositor is endowed with 1 unit of the single good at $t = 0$ and has the following utility function:

$$U(c_1, c_2; \omega_i, \tau) = u(c_1, c_2; \omega_i) + g(\tau) = \frac{(c_1 + \omega_i c_2)^{1-\gamma}}{1-\gamma} + \delta \cdot \frac{\tau^{1-\gamma}}{1-\gamma}$$

A depositor can be either impatient or patient, with preference type $\omega_i \in \Omega = \{0, 1\}$. With probability π, a depositor is impatient ($\omega_i = 0$) and only values consumption c_1 at $t = 1$, whereas with probability $1 - \pi$

she is patient ($\omega_i = 1$) and can consume in either c_1 in $t = 1, 2$. The depositor's coefficient of relative risk aversion satisfies $\gamma > 1$. In addition, each depositor enjoys consumption of τ units of a public good, which is provided by the government. The parameter δ measures the importance of public goods relative to private consumption. The government has an amount T of consumption goods that can be used to provide the public good.

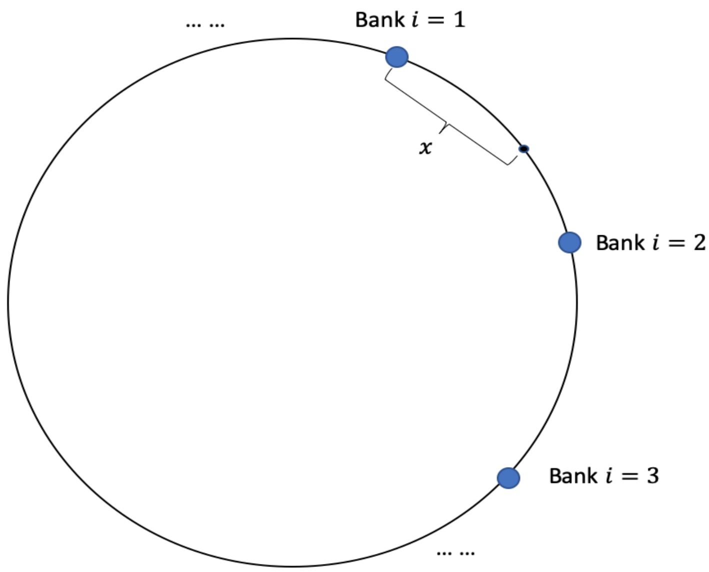

Figure 1. Circular Banking Competition

Bankers can freely enter the market, and each banker operates a single bank. Importantly, each bank incurs an entry cost K at period 0 to operate. I further assume the endogenous number n of banks will be evenly located on the circle, as in Figure 1. All banks possess the same constant-returns-to-scale technology for transforming depositors' endowments into private consumption: One unit of the good deposited in period 0 yields $R > 1$ units in period 2, but only one unit in period 1.

Bankers compete to attract depositors to their banks at $t = 0$. An individual banker i competes with other bankers by choosing a payment schedule $\{c_1^i, c_2^i\}$. This payment schedule determines the bankers' earning per deposit at $t = 0$, which I denote θ^i. The value of θ^i is interpreted as a service fee bank charges each customer at the time of deposit. The banker's total profit depends on this fee θ^i and the number of depositors he can attract. The banker pays the entry cost K at period 0 out of these profits. The free entry condition implies that, in equilibrium, these profits will be just large enough to cover the entry cost. The banker will place the remaining fraction $(1 - \theta^i)$ of deposits into the investment technology to satisfy depositor withdrawals at $t = 1, 2$. All banks serve withdrawals sequentially on a first-come-first-served basis, and deposit contracts are enforced in a standard way in normal times. It is only when a bank run occurs then the government intervenes and determines ex-post depositor payoffs under the bailout policy.

Depositors can choose to deposit in any bank at t=0. However, they must pay a transportation cost of ϕ per unit of distance from their location to the bank. I assume ϕ is small enough such that joining a bank is superior to staying in autarky for all depositors.

2.2 Bank run and Bailouts

At the beginning of $t = 1$, all depositors simultaneously decide whether to withdraw in this period. A depositor j's withdraw strategy can be presented as $y_j : \Omega \to \{0, 1\}$, where $y_j = \{0, 1\}$ corresponds to withdrawal at $t = 1$ and $t = 2$ respectively. A bank run occurs when a positive measure of patient depositors choose to withdraw at $t = 1$. I study systemic bank runs in which depositors rush to withdraw from all banks rather than idiosyncratic runs on an individual bank. What is more, such a crisis is assumed to be unexpected as in Chang and Velasco (2001), Ennis and Keister (2009), and others.

If a bank run occurs, the government can intervene and provide bailouts. If withdrawals do not stop after a fraction π of depositors have withdrawn at $t = 1$, the government realizes a run is underway. Banks will then be placed into a resolution process: the government takes over the banks and aims to maximize depositors' welfare, conditional on banks' remaining resources and its fiscal capacity T. As part of this resolution process, I assume that the run stops and all remaining patient depositors wait to withdraw at $t = 2$, as in Ennis and Keister (2009). The government lacks commitment and, therefore, cannot credibly commit to suspend withdrawals. Instead, it may reallocate part of the public resources T to support the consumption of the remaining depositors who suffer losses.

2.3 Fragility

I study the banking system's fragility by looking at the set of parameter values for which a bank run equilibrium exists. In such an equilibrium, all depositors choose to withdraw at the beginning of period 1, regardless of their types:

$$y_j(\omega_j = 0) = 0 \text{ and } y_j(\omega_j = 1) = 0, \quad \forall j$$

The banking system is said to be *fragile* if this strategy profile is an equilibrium of the withdrawal game. Whether or not this equilibrium exists depends on the parameter values $\{\pi, R, \gamma, \phi, \delta, T\}$ and importantly, on the entry cost K. A change in the level of K will lead to a change in the number of banks n in equilibrium, reflecting a change in the level of banking competition. My primary interest is in how changes in the entry cost K affect the fragility of the banking system. To make this comparison, I will say the banking system is more (less) fragile if the bank run equilibrium exists for a strictly larger (smaller) set of parameter values sustain the bank run equilibrium. The following definition summarizes these.

Definition 1 A change in the entry cost K makes the banking system more (less) fragile if the bank run equilibrium exists for a strictly larger (smaller) set of parameter values $\{\pi, R, \gamma, \phi, \delta, T\}$.

2.4 Timeline

Figure 1 shows the timing of decisions. At $t = 0$, banker i chooses $\{\theta^i, c_1^i, c_2^i\}$. A depositor compares the offers of each bank and decides which bank to join. After depositors join, each banker pays the entry cost K using the fee θ^i charged to depositors and consumes any remaining funds at the end of $t = 0$. At the beginning of $t = 1$, depositors decide when to withdraw and the withdrawing depositors arrive at their banks sequentially to withdraw and consume. If the government sees extra withdrawals after banks have served π fraction of depositors, it steps in with a bailout policy that offers consumption levels $\{\hat{c}_1^i, \hat{c}_2^i\}$. The remaining impatient depositors then withdraw at $t = 1$ and receives $\hat{c}_1^i$. At $t = 2$, the remaining patient depositors withdraw and receive $\hat{c}_2^i$.

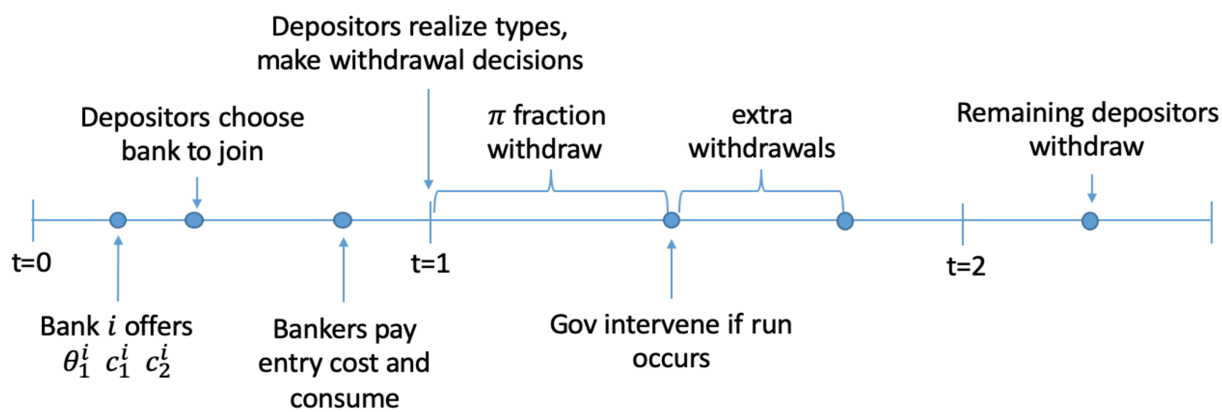

Figure 2. Timeline

2.5 Discussion

An essential feature of my model is that I model competition levels as changes in the entry cost K. This cost represents the entry barrier in the banking sector, and the changes in K naturally corresponds to changes in the level of competition: a low (or zero) cost means little (or no) market barrier with highly competitive market, and vise versa. In section 4, I allow the government to modify K to directly change the competition level in the market. In addition, bankers need to use the fee θ to cover the cost K at $t = 0$. Previous studies that focus on the liability side typically use the realized long term asset return (R) minus the depositors' payoffs to measure the bank's final profit. However, bankers implicitly use their profits to pay depositors as they pay out everything in the event of a bank run. The source of a more stable bank under less competitive environment (like monopoly) comes from the bank's market power to earn more profits. In this paper, I show these results depend on holding the revenues within the bank and there is a different channel that competition amplifies fragility through the government's bailout policy. In sum, competition level in the model is restricted in by K and, as a result, bankers' profits θ cannot be used as a buffer to pay depositors in bank runs.

3 Competition, Bailouts and Fragility

In this section, I derive the conditions for existence of the bank run equilibrium. I show that when bailouts are present, the banking system's fragility increases as the competition level increases. The bailout channel is then turned off to illustrate how my result differs from the traditional buffer channel.

3.1 Competition for Deposits

I start by discussing the circular market shown in Figure 1, in which banks compete for deposits by offering depositor payment schedules in period 0.

A depositor located between bank i and $i+1$ with a distance x to bank i compares two offers from the two neighboring banks: $\{\theta^i, c_1^i, c_2^i\}$ and $\{\theta^{i+1}, c_1^{i+1}, c_2^{i+1}\}$. Note he also sees the offers from banks that are further away but, in equilibrium, the offers will be such that he chooses either bank i or $i+1$. This depositor is indifferent between bank i and $i+1$ when:

$$\pi u(c_1^i) + (1-\pi)u(c_2^i) - \phi \cdot x = \pi u(c_1^{i+1}) + (1-\pi)u(c_2^{i+1}) - \phi \cdot (\frac{1}{n} - x) \tag{1}$$

Banker i maximizes his profit by attracting depositors to deposit in his own bank. He knows a depositor's optimal choice as described above and decides the service fee per depositor, θ^i, based on the amount of deposits he attracts. Note that choosing θ^i generates a trade-off in bank's extensive (number of depositors) and intensive (per depositor profit) margins that determine total profit. For given symmetric neighboring banks offers $\{c_1^{i+1}, c_2^{i+1}\}$, bank i's profit maximization problem is given by:

$$\begin{aligned}
\max_{\theta^i, c_1^i, c_2^i} \quad & \theta^i \cdot [\frac{1}{n} + \frac{\pi u(c_1^i) + (1-\pi)u(c_2^i) - \pi u(c_1^{i+1}) - (1-\pi)u(c_2^{i+1})}{\phi}] - K \\
\text{s.t.} \quad & c_1^i + (1-\pi)\frac{c_2^i}{R} \leq 1 - \theta^i \\
& \pi u(c_1^i) + (1-\pi)u(c_2^i) \geq u(1) + \frac{1}{2n}\phi \\
& 0 \leq \theta^i \leq 1
\end{aligned}$$

where the term in square brackets in the objective function is derived from equation (1), representing the total number of depositors bank i attracts from both sides. The first constraint is the bank's resource constraint. It can be seen that the more the bank i takes (higher θ^i), the fewer depositors the bank can attract given neighboring banks' offers. The second constraint is the incentive constraint that ensures depositors are willing to join the bank rather than remain in autarky. Since ϕ is sufficiently small, this constraint does not bind in equilibrium. In a symmetric equilibrium all banks choose the same payoffs, for now I keep the subscript i and will omit it after describing the bailout policy in the next subsection.

The assumption that ϕ is small enough also ensures that the market is fully covered, i.e. all depositors join a bank in equilibrium. It can be shown the solution to the bank's problem is interior with positive fee $\theta^i > 0$. In particular, c_1^i, c_2^i can written as a function of θ^i and n:

$$\begin{aligned}
c_1^i &= (\frac{\theta^i n}{\phi})^{\frac{1}{\gamma}} \\
c_2^i &= (\frac{\theta^i n R}{\phi})^{\frac{1}{\gamma}} \quad \forall i
\end{aligned}$$

Using these values of c_1^i and c_2^i, the bank's resource constraint gives an equation that implicitly defines a banker's optimal choice of θ^i:

$$\pi \cdot (\frac{\theta^i n}{\phi})^{\frac{1}{\gamma}} + (1-\pi) \cdot (\frac{\theta^i n R}{\phi})^{\frac{1}{\gamma}} = 1 - \theta^i \tag{2}$$

These complete each banker's consumption and offers when competing for deposits in the market. In addition, the number of competing banks n is endogenously determined by C and θ^i in the equilibrium: Bankers will choose to enter the market as long as the revenue is greater than the entry cost K. When the potential revenue is less than K, there will be no more bank entrant as it is costly to enter the market. Therefore, a higher (lower) competition is reflected from a decreasing (increasing) entry cost K with corresponding higher (lower) number of banks competing in the market. The following lemma shows the properties of banks' payments as part of a symmetric equilibrium.

Lemma 1. *As K decreases, the banking sector is more competitive with a larger n. An individual bank i's fee θ^i is smaller while the payoffs c_1^i and c_2^i both increase.*

3.2 Bank Runs and Bailouts

If a bank run occurs, the government learns that a run is underway after a fraction π of depositors have withdrawn. At this point, it will intervene so the individual bankers will cease offering $\{c_1^i, c_2^i\}$ $(i = 1, 2, ..., n)$. For the remaining $(1-\pi)$ depositors, a measure $(1-\pi)\pi$ of them are impatient and the remaining $(1-\pi)^2$ are patient. The government may find it ex-post optimal to reallocate part of its resources to mitigate these depositors' losses. It chooses the optimal bailout payment b_i for $i = 1, 2, ...n$ and rearranges payoffs $\{\hat{c}_1^i, \hat{c}_2^i\}$ for each depositor at bank i to meet the demand for extra withdrawals. The ex-post welfare maximization problem of the government is:

$$\max_{b_i, \hat{c}_1^i, \hat{c}_2^i, i=1,2..n} \quad \frac{1}{n}\sum_{i=1}^{n}(1-\pi)[\pi u(\hat{c}_1^i) + (1-\pi)u(\hat{c}_2^i)] + g[T - \bar{b}]$$

$$\text{s.t.} \quad (1-\pi)[\pi\hat{c}_1^i + (1-\pi)\frac{\hat{c}_2^i}{R}] = 1 - \theta^i - \pi c_1^i + b_i$$

$$\bar{b} = \frac{1}{n}\sum_{i=1}^{n} b_i$$

$$b_i \geq 0 \ \ \forall i$$

where $g(\tau) = \delta \cdot \frac{\tau^{1-\gamma}}{1-\gamma}$. Since there is one unit of depositor in total, $\bar{b}$ is also interpreted as the average bailout package per depositor.

Two cases can emerge when taking the first order condition to find the optimal b_i. First, when the amount of public goods T is relatively abundant, there are interior solutions b_i for all $i = 1, 2, ..., n$. The optimal bailout package b_i and the rearranged payoffs $\hat{c}_1^i$, $\hat{c}_2^i$ satisfy:

$$g'(T - \bar{b}) = u'(\hat{c}_1^i) = Ru'(\hat{c}_2^i), \quad \forall i \tag{3}$$

This condition states the marginal return from public consumption and from private consumption of the remaining should be equal. Second, if the amount of public goods T is sufficiently small, it can be the case that the government will provide no bailouts. The public goods' marginal utility will be so high such that $g'(T) > u'(\hat{c}_1^i) = Ru'(\hat{c}_2^i)$ with $b_i = 0$ thus $\bar{b} = 0$. This occurs when the fiscal capacity is relatively small and the non-negativity constraint on b_i binds. In particular, it satisfies $T < \underline{T}$, where $\underline{T}$ is solved from taking $\theta = 0$ (fully competitive) and it is the smallest fiscal capacity that the government will choose to provide bailouts. A no-bailout policy is a special case with $\bar{b} = 0$ and I will show that it leads to a more fragile system. Thus, it suffices to focus on the first case to show the bailout amplification channel with

$$T > \underline{T} = \delta^{\frac{1}{\gamma}} \frac{(1-\pi)R^{\frac{1}{\gamma}-1}}{[\pi + (1-\pi)R^{\frac{1}{\gamma}-1}]^2}$$

Condition (3) above suggests two things. First, the government views all the depositors across banks equally and optimizes the bailout package by providing the payoffs $\{\hat{c}_1^i, \hat{c}_2^i\}$ for all $i = 1, 2, ...n$ banks' depositors. Second, $\hat{c}_1^i < \hat{c}_2^i$ is satisfied, that is, the remaining patient depositors consume more than the remaining patient depositors. Together with the ex-ante bank's symmetric payoffs, the depositor's complete payoff schedule can be presented as $\{c_1, c_2, \hat{c}_1, \hat{c}_2\}$ $\forall i$. Hereafter I drop the subscript i on banks and the payoffs.

From lemma 1, the payments offered to depositors by each bank increase in n. It turns out the ex-post payments $\{\hat{c}_1, \hat{c}_2\}$ also increase in competition, even though more resources are paid out by banks ex-ante. This is because the bank's fee θ decreases more than the increase in c_1. This fact can be seen from the first resource constraint in the bank's problem. The resource constraint always binds and the decrease in θ contributes to the increase in both c_1 and c_2. Thus more resources are left in the bank to pay for the c_2 late withdrawals. The rising ex-ante payoff c_2 contributes to higher ex-post payoffs as well. The following lemma summarises this result.

Lemma 2. *The symmetric ex-post payoffs satisfy $\hat{c}_1 < \hat{c}_2$. Both payoffs increase as competition level increases (lower K).*

3.3 Equilibrium and Fragility

Based on these results, we can find the symmetric equilibrium at certain competition level measured by K. For a given parameter set $\{\phi, \sigma, \delta, T, R\}$, an individual banker maximizes his profit by solving the bank's problem. Depositors join the neighboring bank that offers the highest expected utility. If a bank run occurs, the government best responds by solving the government's problem, and depositors receive $\{\hat{c}_1, \hat{c}_2\}$. Importantly, there may be multiple symmetric equilibria in the system. There is one symmetric no bank run equilibrium in this economy. Meanwhile, there could be another symmetric bank run equilibrium, in which all depositors run the banks and the government may provide bailouts to optimize the ex-post payoffs. Define f, which I refer to as the fragility ratio, by:

$$f = \frac{c_1}{\hat{c}_2}$$

This ratio represents the relative size of the bank's early payoff to the rearranged late payoff. Using the entry cost K as a measure of competition level, I show the condition for the existence of the symmetric bank run equilibrium in the following proposition.

Proposition 1. *Given competition level K, there is a symmetric no bank run equilibrium without government intervention. In addition, there is a symmetric bank run equilibrium when $f > 1$. If $T \geq \underline{T}$, the government provides bailouts to depositors in the run equilibrium.*

Recall the free-entry condition with entry cost K pins down the number n of banks in the economy. Banks choose the same positive fee level θ per deposit at the symmetric equilibrium. A depositor will join the closest bank to her location. If she is located precisely in the middle of two banks, she is indifferent between patronizing either bank. It is easy to see there is one symmetric no bank run equilibrium. At the beginning of period 1, an individual depositor's type ω_j is privately realized. If she is impatient ($\omega_j = 0$) she will always withdraw at $t = 1$. If she is patient ($\omega_j = 1$), she compares the payoffs from withdrawing in period 1 and wait for period 2. If all other patient depositors wait to withdraw in period 2, she will also wait to consume c_2 since the bank's payoffs satisfy $c_1 < c_2$ without triggering bailouts.

In addition, there may also be a bank run equilibrium. If all other patient depositors run at period 1, an individual patient depositor's incentive to run is determined by the relative size of c_1 and the rearranged late payoff $\hat{c}_2$. In this case, the bailout policy is triggered because there are more than π fraction of early withdrawals. The patient depositor will receive $\hat{c}_2$ instead of c_2 if she waits. And if $\hat{c}_2 < c_1$, the patient depositor will also join the run id she is not among the first π fraction when arriving at the bank, the patient depositor will return and wait to withdraw at period 2 since $\hat{c}_1 < \hat{c}_2$. Thus, $c_1 > \hat{c}_2$ is the only source of incentive for the patient depositor to withdraw early. In other words, if the fragility ratio $f > 1$, there is a symmetric bank run equilibrium in which all patient depositors will choose to withdraw early.

Now I turn to analyze the impact of competition by modifying the entry cost K. As K changes, competition affects the number of banks n and the size of banking sector relative to the government's fiscal capacity. These further lead to changes in the bailout payment per depositor in the economy. In particular, a decreasing entry cost K leads to a higher n and a lower θ at the equilibrium. This lower θ corresponds to the higher depositors payoffs $\{c_1, c_2, \hat{c}_1, \hat{c}_2\}$, and the rising ex-ante payoffs $\{c_1, c_2\}$ corresponds to a rise in the size of each individual bank's liabilities. For the total one unit of deposit in the market, the increase in $\{c_1, c_2\}$ can also be interpreted as larger total liabilities issued in the banking system due to higher competition.

At first glance, the larger size banking system seems in need of a larger bailout. However, the optimal bailout amount decreases since depositors have higher consumption levels, as can be seen from condition (3). At the equilibrium, condition (3) holds with equality. Higher $\hat{c}_1$ and $\hat{c}_2$ at a higher competition level requires a smaller bailout size, so that more resources remain to provide public goods. Providing bailouts tends to become more expensive in terms of providing public goods. The average bailout package per depositor $\bar{b}$ thus decreases in n. This is the key channel how increased competition affects fragility through bailout policy.

More importantly, the rising payoffs and smaller bailouts make the banking system more fragile, as the run equilibrium exists for a larger set of parameter values.

Proposition 2. *As competition level increases (lower K), the bailout package per depositor $\bar{b}$ decreases and the banking system becomes more fragile.*

The first bailout result comes directly as discussed above. The bailout package $\bar{b}$ per depositor decreases because when competition rises, providing bailout becomes more expensive at the cost of providing public goods. Better-treated depositors mean fewer limited public resources should be diverted to provide bailouts relative to providing public goods. Although more resources have been paid out during the first π fraction withdrawals, $\bar{b}$ decreases.

It is less clear how the rising competition affects financial fragility. Denote the fragility ratio f under the bailout policy as f_b. We can take a look at the components of the ratio f_b, $\hat{c_2}$ and c_1. From the government's problem, I can replace $\hat{c_1}$ and $\bar{b}$, then represent $\hat{c_2}$ as a function of c_1:

$$\hat{c_2} = \frac{1 - \theta - \pi c_1 + T}{(1-\pi)[\pi R^{-\frac{1}{\gamma}} + (1-\pi)R^{-1}] + (\frac{\delta}{R})^{\frac{1}{\gamma}}}$$

Meanwhile, instead of using $c_1 = (\frac{\theta n}{\phi})^{\frac{1}{\gamma}}$, c_1 can be equivalently expressed as a function of θ solely:

$$c_1 = \frac{1-\theta}{\pi + (1-\pi)R^{\frac{1}{\gamma}-1}}$$

This expression comes from using the bank's resource constraint and replacing c_2. Note that c_1 and $\hat{c}_2$ are both increasing as competition increases, as the decreasing entry barrier K contributes to a lower equilibrium fee θ and higher depositor payments. Nevertheless, Proposition 2 suggests that the decreasing bailout package $\bar{b}$ has a dominant effect on raising depositor's incentive to run. The increasing competition level leads to a slower increase in rearranged payoff $\hat{c}_2$ relative to early payoff c_1. In particular, fragility ratio f_b can be written as:

$$f_b = \frac{c_1}{\hat{c}_2} = \frac{1-\theta}{1-\theta-\pi c_1 + T} \cdot \bar{A} \tag{4}$$

where $\bar{A} = \frac{(1-\pi)[\pi R^{-\frac{1}{\gamma}} + (1-\pi)R^{-1}] + (\frac{\delta}{R})^{\frac{1}{\gamma}}}{\pi + (1-\pi)R^{\frac{1}{\gamma}-1}}$, which does not depend on θ. Then f_b can be solely represented as a function of θ after replacing c_1.

It has been shown $\frac{d\theta}{dK} > 0$, i.e. higher competition (lower entry cost K) leads to a lower fee θ at the equilibrium. Meanwhile, the equation above shows the depositors' payments are affected by K only through θ. So I can apply the chain rule and get $\frac{df}{dK} = \frac{df}{d\theta} \cdot \frac{d\theta}{dK}$. The remaining work is to examine the sign of

$\frac{df}{d\theta}$, and it turns out to satisfy $\frac{df}{d\theta} < 0$. I provide the details of the derivation in the Appendix. Thus, $\frac{df}{dK} = \frac{df}{d\theta} \cdot \frac{d\theta}{dK} < 0$. As the entry cost decreases, competition increases and banks charge lower fees. The lower θ and amplifies the disproportional increase in payoffs by increasing c_1 faster than $\hat{c}_2$. The system is more fragile as the parameter set $\{\pi, R, \gamma, \phi, \delta, T\}$ for which run equilibrium exists, i.e. $f_b > 1$, is larger.

Intuitively, depositors recognize that the bailout package $\bar{b}$ would be smaller under higher competition levels. Their incentive to run and attempt to withdraw before banks are placed in resolution is higher when banks face more competition and depositors anticipate bailouts will be smaller. When competition increases, the payoff schedule $\{c_1, c_2, \hat{c}_1, \hat{c}_2\}$ will have a larger parameter set $\{\pi, R, \gamma, \phi, \delta, T\}$ that falls into the region $c_1 > \hat{c}_2$. The run equilibrium exists in this larger parameter set. Thus the banking system is more fragile.

Figure 3 below gives an example of the result in Proposition 2. It illustrates how f_b increases with competition level from the perspective of banks amount n. Recall that $\frac{dn}{dC} < 0$, the increasing competition corresponds to the rising competing banks in the market. The shaded areas show an expansion of the parameter space that holds the run equilibrium, under the dimensions of R (the long-term return) and π (the fraction of impatient depositors), given other parameters[2]. The boundary lines are drawn as $f_b = 1$ for each n. When K decreases, the ratio f_b increases in n for given $\{\pi, R, \gamma, \phi, \delta, T\}$ so the parameter set that sustains $f_b \geq 1$ is strictly larger. Notice the fragile regime may not be monotone in some parameters. For instance, when $\pi = 0.46$ and $n = 100$, the system is not fragile when R is either very big or small, but is fragile with middle size $R = 1.3$. Such non-monotone feature is consistent with Li (2017), who showed the fragility varies in a non-monotone pattern with respect to the return on assets in a competitive banking system.

[2]In this example, the other parameters are chosen as: $\gamma = 3, \phi = 0.01, \delta = 1$ and $T = 1$

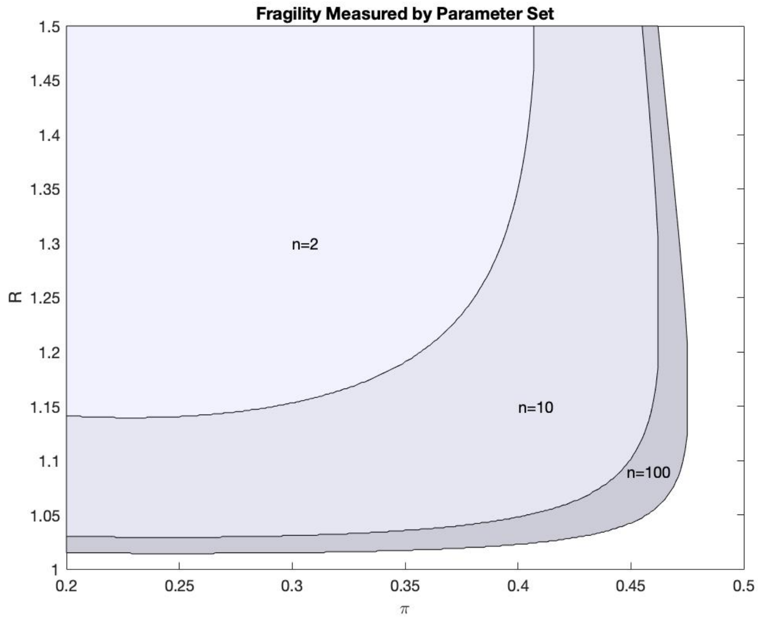

Figure 3. Fragility Measured by Parameter Set

Propositions 2 has two more implications. First, the model includes the fully competitive and monopolistic banking system as two extreme cases. The competitive system is most fragile because it leads to the largest bank liabilities relative to the bailout capacity and the highest f_b. On the other hand, a monopolistic bank is the other extreme, with the lowest f_b. In particular, a monopolist can pay less without losing depositors comparing to a duopoly system. Corollary 1 suggests that depositors' utilities are above autarky plus transportation costs in the duopoly system. Thus a monopolistic banking system generates the lowest payoffs and the corresponding lower fragility.

Second, the channel by which competition amplifies fragility through bailouts is likely to appear even when the ex-ante fiscal capacity T is not fixed but instead chosen by the government at $t = 0$, as in Keister (2016). Treating T as fixed simplifies the model and illustrates the mechanism at work in the clearest way. In particular, it shows how the driving force is the relative sizes of banking sector liabilities (c_1, c_2) and the government's fiscal capacity. In fact, condition (3) suggests that as long as T does not increase at the same speed as θ shrinks, an increase in competition will increase fragility.

3.4 Fragility with no bailouts

To see how this bailout channel is different from traditional ones, I can turn off the bailout policy in the model to see whether competition still leads to a more fragile system. Note this result is equivalent to when the public resource is small, i.e. $T < \underline{T}$, and the government optimally choose no bailouts. Without bailouts, banks still compete to attract depositors in the circular market. The only difference is that now, when the bank run occurs, the average bailout package $\bar{b} = 0$. In this case, I can show competition level n still affect bank's fee θ and payments $\{c_1, c_2, \hat{c}_1, \hat{c}_2\}$, but it no longer changes fragility.

Proposition 3. *Without bailouts, banks' equilibrium fee θ increases in K while depositors' equilibrium payments $\{c_1, c_2, \hat{c}_1, \hat{c}_2\}$ all decrease in K. Bank's fragility remains unchanged as K changes.*

Denote the fragility ratio without bailouts as f_{nb}, which can be written as:

$$f_{nb} = \frac{c_1}{\hat{c}_2} = [\pi R^{1-\frac{1}{\gamma}} + (1-\pi)] \cdot R^{-\frac{1}{\gamma}} \tag{5}$$

The results follow directly from this expression, which does not depend on the entry cost K.

The difference here from the previous section is that the fragility measure f_{nb} no longer depends on bailout policy, so the tradeoff between public goods and bailouts disappears. The payoffs $\{c_1, c_2, \hat{c}_1, \hat{c}_2\}$ are increasing as the entry cost K decreases. However, the payoffs change in K at the same speed, so the fragility measure f_{nb} holds constant. It is not surprising that a more competition n necessarily means depositors' utilities are higher. However, this does not necessarily induce a higher incentive to run, as the relative amount between early and late payoffs remains unchanged. From bank's perspective, rising competition naturally shrinks revenue. But in the absence of bailouts, competition does not contributes to a more fragile system through affecting depositors' incentive to run on the banking system. This with-without bailouts comparison highlights the importance of assumptions about banks' capital structure in modeling. If profits are held in the bank and used to pay depositors in the event of a run, financial fragility will naturally tend to decrease when bank profits are larger.

4 Policy Implications

Given that competition can increase financial fragility, it is natural to ask: should banking competition be restricted? This section discusses two policies that aim mitigate fragility in the framework of my model: restricting competition and imposing an interest rate ceiling. While both policies help to reduce fragility, I show that the interest ceiling policy does so at a lower cost.

Suppose the government wants to reduce financial fragility by a given amount. It can restrict competition by directly raising the entry cost K, which will cause the equilibrium number of banks to decrease. Let n_0 be the number of banks under the original entry cost, and n_1 be the number of banks at the new, higher entry cost. I also denote the former's fragility ratio as f_{high}, while the latter targeted lower fragility ratio as

f_{low}. The reduction in financial fragility from $\Delta f = f_{high} - f_{low}$, or equivalently n_0 to n_1, can be achieved by raising K in the market. To compare the effectiveness of the two policies, I compare the welfare changes induced by this same Δf change in financial fragility under each policy.

Denote depositors' equilibrium payments under the original entry cost as $\{c_1^*, c_2^*, \hat{c}_1^*, \hat{c}_2^*\}$ and under the new, higher entry cost as $\{c_1^r, c_2^r, \hat{c}_1^r, \hat{c}_2^r\}$. From Proposition 2, the payoffs satisfy $c_t^* > c_t^r$ and $\hat{c}_t^* > \hat{c}_t^r$ for $t = 1, 2$ because at the equilibria, the number of banks satisfies $n_0 > n_1$. Restricting competition is costly for depositors especially because the fewer banks charge higher fee θ. Let depositor's welfare for each entry cost denoted W_0 and W_1. Depositors' ex ante expected welfare clearly has $W_0 > W_1$ with

$$W_0 = \pi u(c_1^*) + (1-\pi)u(c_2^*) + g(T)$$
$$W_1 = \pi u(c^r) + (1-\pi)u(c_2^r) + g(T)$$

respectively. Note that, since the en-ante probability of a crisis is negligible, welfare here only depends on c_1 and c_2.

The second policy to lower fragility is to impose a cap on early payment c_1, so the depositor's incentive to run is restricted. Denote this cap as $\bar{c}$. Government is able to choose this $\bar{c}$ to achieve its financial stability goal, while the banks can still compete to offer c_2 for deposits. Proposition 4 below characterizes how the remaining payments $\{c_2, \hat{c}_1, \hat{c}_2\}$ and the number of banks n change in $\bar{c}$ at the equilibrium.

Proposition 4. *In the symmetric equilibrium as $\bar{c}$ decreases, the payments $\{c_2,\ \hat{c}_1,\ \hat{c}_2\}$ all increase, the fee θ increases and the competing banks n increases.*

I denote depositor's welfare under the interest rate ceiling policy as W_2 when the lower fragility f_{low}, is reached. And denote the payoffs as $\{\bar{c}^e, c_2^e, \hat{c}_1^e, \hat{c}_2^e\}$, the number of banks at the equilibrium as n_2. Then W_2 can be written as

$$W_2 = \pi u(\bar{c}^e) + (1-\pi)u(c_2^e) + g(T)$$

Under the interest ceiling policy, banks compete more on c_2 by offering $c_2^e > c_2^*$ when the ceiling is imposed with $\bar{c}^e < c_1^*$. More importantly, compared to raising the entry cost C, the following lemma shows that the payoffs of the two policies satisfy $\bar{c}^e > c_1^r$ and $\hat{c}_2^e > \hat{c}_2^r$.

Lemma 3. *For Δf changes, $\bar{c}^e > c_1^r$ and $\hat{c}_2^e > \hat{c}_2^r$.*

The targeted fragility ratio at f_{low} can be expressed as:

$$f_{low} = \frac{\overline{c}^e}{\hat{c}_2^e} = \frac{c_1^r}{\hat{c}_2^r}$$

When the government chooses the interest ceiling rate $\bar{c}$ ($< c_1^*$) to lower fragility, from Proposition 4 we know the late payoff $\hat{c}_2^e$ is increasing as $\bar{c}$ decreases. This increasing $\hat{c}_2^e$ makes the targeted lower fragility ratio f_{low} to be achieved faster, comparing to the restricting competition policy that raises K. The increase in K contributes to the lower amount in both c_1 and c_2. The different payoff responses of these two policies contributes to different effectiveness in reducing financial fragility. In addition, it can also be seen that the number of competing banks in the interest rate ceiling policy, n_2, should be larger or equal[3] to n_0, which is the number of banks before implementing the policy. This is because the lowering $\bar{c}$ leaves room for both higher θ and c_2^e: bankers charge higher fees, despite the fact that they also make higher payments to depositors. By imposing restriction on only the early payment, the government can achieve its goal of lowering financial fragility at a lower cost on depositors.

Proposition 5. *The interest rate ceiling policy can achieve a given reduction in financial fragility at a lower welfare cost than restricting competition.*

For a given fragility change Δf, we always have $W_0 > W_2 > W_1$. In other words, restricting competition is less effective as it leads to more losses on depositors welfare. Note that n_2 is also greater or equal to n_1, which suggests that the interest ceiling policy actually leads to a relatively more competitive market than restricting competition. The interest rate ceiling policy allows competition to remain to play a role, banking competition favors depositors and is more efficient.

In sum, restricting competition is inefficient as it creates more payoff distortions and generates higher welfare losses for depositors. When bailouts create the “too big to save” scenario that amplifies fragility, an interest rate ceiling policy is always more effective than reducing banking competition.

5 Conclusion

In this paper, I study the relationship between banking competition and financial fragility, focusing on the liability side of bank balance sheets. I introduce spatial competition into a Diamond-Dybvig (1983) bank run model with limited commitment and bailouts. Competition amplifies financial fragility through the prevailing bailout policy while inducing banks to offer depositors higher payoffs. Having an additional bank in the system raises depositors’ welfare in normal times, but makes the bailout per depositor smaller in the

[3] Since I focus on n being integers, the fee change may be small enough to change n.

event of a crisis. If a crisis occurs, depositors' incentives to run would be higher in a more competitive banking system, as relatively fewer bailout resources per depositor are available. It is harder to save a larger banking system, which makes the banking system more fragile. In this sense, a "too big to save" problem arises. This relation holds even though the traditional link, which depends on using profits as buffers, disappears. My results highlight the importance of capital structure in modeling banking fragility. In addition, I also compare two policies, an interest rate ceiling and restricting competition by increasing the entry costs, both of which can enhance financial stability. Restricting competition cannot be optimal as it generates large welfare losses on depositors. On the other hand, an interest rate ceiling policy restricts the incentive to run more effectively by letting competition continue to play a role.

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Proofs

1. Proof of Lemma 1

In bank's problem, it can be first seen that θ^i cannot be 0 or 1, otherwise the bank makes no revenue and has negative profit. Then, with the remaining first two constraints, I take the first order condition with respect to θ^i, c_1^i and c_2^i and get:

$$[\frac{1}{n} + \frac{\pi u(c_1^i) + (1-\pi)u(c_2^i) - \pi u(c_1^{i+1}) - (1-\pi)u(c_2^{i+1})}{\phi}] - \mu_1 = 0$$
$$\theta\frac{\pi}{\phi}(c_1^i)^{-\gamma} - \pi\mu_1 + \pi(c_1^i)^{-\gamma}\mu_2 = 0$$
$$\theta\frac{1-\pi}{\phi}(c_2^i)^{-\gamma} - \frac{1-\pi}{R}\mu_1 + (1-\pi)(c_2^i)^{-\gamma}\mu_2 = 0$$

where μ_1 and μ_2 are the multipliers for the first and second constraint in the bank's problem. Imposing symmetry in payoffs, the three conditions can be simplified as:

$$\frac{1}{n} = \mu_1$$
$$\frac{\theta}{\phi}(c_1)^{-\gamma} = \mu_1 - (c_1)^{-\gamma}\mu_2$$
$$\frac{\theta}{\phi}(c_2)^{-\gamma} = \frac{1}{R}\mu_1 - (c_2)^{-\gamma}\mu_2$$

For small enough transportation cost ϕ, the second incentive constraint does not bind, so $\mu_2 = 0$. Thus, the solutions can be solved from the three conditions:

$$c_1^i = (\frac{\theta n}{\phi})^{\frac{1}{\gamma}}$$
$$c_2^i = (\frac{\theta nR}{\phi})^{\frac{1}{\gamma}}$$
$$1-\theta = \pi\cdot(\frac{\theta n}{\phi})^{\frac{1}{\gamma}} + (1-\pi)\cdot(\frac{\theta nR}{\phi})^{\frac{1}{\gamma}}$$

Note the third resource constraint suggests $\frac{d\theta}{dn} < 0$ holds. In addition, the bank's total profit Π can be expressed as:

$$\Pi = \frac{\theta}{n} - K$$

At the equilibrium $\Pi = 0$, the number of banks in the economy equals $n = \frac{\theta}{K}$ as an implicit function of θ.

When K increases, the number of banks n in the equilibrium will decrease. This result can be proved by contradiction: if n increases in K, $\frac{d\theta}{dn} < 0$ suggests θ decreases. However, this leads to $\Pi = \frac{\theta}{n} - K < 0$, a contradict to the equilibrium zero profit condition.

2. Proof of Proposition 2

1) Properties of $\hat{c_1}, \hat{c_2}, \bar{b}$

Imposing symmetry in the government's problem, it is equivalent to solve for the optimal $\{\hat{c}_1.\hat{c}_2, \bar{b}\}$. Thus, the government's problem is:

$$\begin{aligned} \max_{\bar{b},\hat{c}_1,\hat{c}_2} \quad & \frac{1}{n}\sum_{i=1}^{n}(1-\pi)[\pi u(\hat{c}_1)+(1-\pi)u(\hat{c}_2)]+g[T-\bar{b}] \\ \text{s.t.} \quad & (1-\pi)[\pi\hat{c}_1+(1-\pi)\frac{\hat{c}_2}{R}] \leq 1-\theta-\pi c_1+\bar{b} \\ & \bar{b} \geq 0 \end{aligned}$$

Let $\hat{\mu_1}$, $\hat{\mu_2}$ be multipliers for the first and second constraint. The first order conditions and non-negativity constraints are:

$$\begin{aligned} u'(\hat{c}_1)-\hat{\mu_1} &= 0 \\ u'(\hat{c}_1)-\frac{\hat{\mu_1}}{R} &= 0 \\ -g'(T-\bar{b})+\hat{\mu_1}+\hat{\mu_2} &= 0 \\ \hat{\mu_1}(1-\theta-\pi c_1+\bar{b}-(1-\pi)[\pi\hat{c}_1+(1-\pi)\frac{\hat{c}_2}{R}]) &= 0 \\ \hat{\mu_2}\cdot\bar{b} &= 0 \end{aligned}$$

It is clear $\hat{\mu_1} > 0$, but there are two cases for the bailout payment $\bar{b}$: $\bar{b} > 0$ and $\bar{b} = 0$. In the first case, $\bar{b} > 0$ and $\hat{\mu_2} = 0$. This gives the optimal interior solution $\bar{b}$. The marginal utility between public goods and payoffs $\hat{c}_1$, $\hat{c}_2$ should be equal:

$$g'(T-\bar{b}) = u'(\hat{c}_1) = Ru'(\hat{c}_2)$$

In this case, the government's rearranged payoffs $\{\hat{c_1}, \hat{c_2}\}$ as a function of en-ante payoffs $\{c_1, c_2\}$ are:

$$\begin{aligned} \hat{c}_1 &= R^{-\frac{1}{\gamma}}\cdot\hat{c}_2 \\ \hat{c}_2 &= \frac{1-\theta-\pi c_1+T}{(1-\pi)[\pi R^{-\frac{1}{\gamma}}+(1-\pi)R^{-1}]+(\frac{\delta}{R})^{\frac{1}{\gamma}}} \end{aligned}$$

while $c_1 = (\frac{\theta n}{\phi})^{\frac{1}{\gamma}}$ from the ex-ante bank's problem. Since $\{c_1, c_2\}$ are increasing in competition level measured by C, it is clear $\{\hat{c_1}, \hat{c_2}\}$ are also increasing in C. And $\bar{b}$ is decreasing.

Note the analysis above relies on interior solution $\bar{b}$. There could be another case that $\bar{b}$ binds on $\bar{b} = 0$:

$$g'(T) \geq u'(\hat{c}_1) = Ru'(\hat{c}_2)$$

This case holds when the public resources T is too small. The marginal utility from public goods is large enough so that it is not optimal to provide any bailouts. In the main context, I assume the public resources T should be relatively large ($T > \underline{T}$) to rule out this less interesting possibility. But here I show the exact condition that $\underline{T}$ satisfies.

Note that, this second binding case is more likely to hold when competition level is high (lower C and higher n). As C decreases the number of competing banks n increases. Meanwhile, $\hat{c}_1$ and $\hat{c}_2$ increase, the marginal utilities from these bailout payments are lower relative to $g'(T)$. Then for $C \to 0$, define the most competitive crisis ex-post bailout payments as $\bar{c}_1^*$ and $\bar{c}_2^*$, if the inequality holds as:

$$g'(T) < u'(\bar{c}_1^*) = Ru'(\bar{c}_2^*)$$

then this optimization will always generates interior solution $\bar{b} > 0$. It means even though depositors are treated very well with $\bar{c}_1^*$ and $\bar{c}_2^*$, it is still optimal to provide bailouts so that there is interior solutions. Define this lower bound T for interior solution as $\underline{T}$, I can get its explicit form from the most competitive environment from:

$$g'(\underline{T}) = u'(\bar{c}_1^*)$$

where $\bar{c}_1^*$ is solved as $\theta = 0$ and $\bar{b} = 0$. So $\underline{T} = \delta^{\frac{1}{\gamma}} \frac{(1-\pi)R^{\frac{1}{\gamma}-1}}{[\pi+(1-\pi)R^{\frac{1}{\gamma}-1}]^2}$.

2) Derive $\frac{df}{dK} < 0$

Now turn to look at banking fragility measured by the existence of the bank run equilibrium. As K changes, the parameter set that satisfies $f > 1$ also changes. From the government's problem, I can replace $\hat{c_1}$ and $\bar{b}$, then represent $\hat{c_2}$ as a function of c_1:

$$\hat{c_2} = \frac{1 - \theta - \pi c_1 + T}{(1-\pi)[\pi R^{-\frac{1}{\gamma}} + (1-\pi)R^{-1}] + (\frac{\delta}{R})^{\frac{1}{\gamma}}}$$

Meanwhile, instead of using $c_1 = (\frac{\theta n}{\phi})^{\frac{1}{\gamma}}$, c_1 can be equivalently expressed as a function of θ solely:

$$c_1 = \frac{1-\theta}{\pi + (1-\pi)R^{\frac{1}{\gamma}-1}}$$

This comes from using bank's resource constraint and replacing c_2. So the fragility ratio f can be written as:

$$f = \frac{c_1}{\hat{c}_2} = \frac{1-\theta}{1-\theta-\pi c_1+T} \cdot \bar{A}$$

where $\bar{A} = \frac{(1-\pi)[\pi R^{-\frac{1}{\gamma}}+(1-\pi)R^{-1}]+(\frac{\delta}{R})^{\frac{1}{\gamma}}}{\pi+(1-\pi)R^{\frac{1}{\gamma}-1}}$, does not depend on θ.

It has been shown $\frac{d\theta}{dK} > 0$, i.e. higher competition (lower barrier K) leads to lower fee θ per unit of deposit. Meanwhile, the payments are affected by K only through θ, either directly through θ or indirectly through n. Thus, apply chain rule I get $\frac{df}{dK} = \frac{df}{d\theta} \cdot \frac{d\theta}{dK}$. The remaining work is to examine the sign of $\frac{df}{d\theta}$:

$$\begin{aligned}\frac{df}{d\theta} &= \frac{\bar{A}}{(1-\theta-\pi\bar{c}+T)^2} \cdot \{-[1-\theta-\pi\cdot\frac{1-\theta}{\pi+(1-\pi)R^{\frac{1}{\gamma}-1}}+T]-(1-\theta)[-1+\frac{\pi}{\pi+(1-\pi)R^{\frac{1}{\gamma}-1}}]\} \\ &= \frac{\bar{A}}{(1-\theta-\pi\bar{c}+T)^2} \cdot \{-(1-\theta)+\frac{\pi(1-\theta)}{\pi+(1-\pi)R^{\frac{1}{\gamma}-1}}-T+(1-\theta)-\frac{\pi(1-\theta)}{\pi+(1-\pi)R^{\frac{1}{\gamma}-1}}\} \\ &= -\frac{\bar{A}\cdot T}{(1-\theta-\pi\bar{c}+T)^2} \\ &< 0\end{aligned}$$

The effect of changing θ got cancelled out in denominator and $\frac{df}{d\theta} < 0$. Thus, $\frac{df}{dK} = \frac{df}{d\theta} \cdot \frac{d\theta}{dK} < 0$. As the entry cost decreases, competition increases and the fees charged by banks decrease. The decrease in θ amplifies the disproportional increase in payoffs by increasing c_1 faster than $\hat{c}_2$. The system is more fragile as the parameter values that holds the run equilibrium, i.e. $f > 1$, is larger.

3. Proof of Proposition 4

Under the interest ceiling policy, bank's ex-ante late payment c_2^e can also be expressed as:

$$c_2^e = (\frac{\theta n R}{\phi})^{\frac{1}{\gamma}}$$

So the fee θ can be solved from:

$$\pi\bar{c} + \frac{1-\pi}{R} \cdot (\frac{\theta n R}{\phi})^{\frac{1}{\gamma}} = 1-\theta$$

At the equilibrium, free-entry condition ensures $\frac{\theta}{n} = K$. Replace n in the above equation, it can be seen as the interest ceiling cap $\bar{c}$ decreases to lower fragility ratio, θ increases at the new equilibrium. Since K holds constant, the equilibrium number of banks n also increases.