Privacy, Payments and Bank Stability

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Abstract

I study how lenders' access to the information contained in borrowers' payments data affects financial stability and welfare. When lenders can infer more about borrower quality, they are able to discontinue investment projects with low pledgeable returns. Doing so harms borrowers whose projects have high non-pledgeable returns. By offering privacy, a central bank digital currency (CBDC) would facilitate risk-sharing between borrowers and lenders. At the same time, however, a private means of payment like CBDC will affect equilibrium interest rates and banks' portfolio choices. I show that privacy leads banks to hold more liquid asset portfolios and thereby increases financial stability. In equilibrium, borrowers with non-pledgeable returns benefit from privacy in payments but depositors are worse off. The central bank can offset some of these effects by providing payments information back to lenders.

Keywords: Privacy, Anonymous Payment, Central Bank Digital Currency, Bank Stability

JEL Classification: E22, E61, G21, G28

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1 Introduction

Payment data generates valuable information and brings concerns about privacy. Consumers' payment histories reveal their preferences (Garratt and Van Oordt (2021)), and their digital footprints can be used by banks to predict their default risk (Berg et al. (2020), Parlour et al. (2020)). Borrowers may also be concerned about their private information being used against their interests. For example, entrepreneurs who borrow from banks may worry about their loans not being renewed base on information from payments data. As advancing computer technology makes it easier to extract information from payment data and leaves privacy more vulnerable, the privacy issue in payment becomes more urgent. Such privacy concerns create a potential role for a form of anonymous payment that keeps the users' information private. In response, the central banks are actively considering to offer central bank digital currency (CBDC), which could fulfill such privacy needs. Compared with cash, it could be widely and conveniently used for payment transactions. The payment information would also accrue to the central bank, which has no incentive for making profits. As Lagarde (2018) stated:

"This (central bank digital) currency could satisfy public policy goals, such as (i) financial inclusion, (ii) security and consumer protection; and to provide what the private sector cannot: (iii) ***privacy in payments****."*[1]

However, anonymous forms of payment may adversely affect banks. As cash usage has declined, banks handle the majority of payment activities in the economy. These activities create information that is useful monitoring loans and managing assets. Bringing in another widely used payment method like CBDC may reduce the information available in the current banking system, a concern expressed in the European Central Bank (ECB)'s report:

"If banks decrease their role in deposit-taking and intervene less in the routing of payment instructions, they might have less information about clients, which, in turn, would harm their risk assessment capacity"[2].

Despite this concern, the macroeconomic implications of such information loss are unclear. In this paper, I show how a privacy-preserving payment option impacts the lending market through bank's portfolio choices, financial stability, and welfare.

I construct a model in which a group of entrepreneurs have heterogeneous incentives for privacy. The traditional informative payment method (which I call 'debit card payment') cannot keep their production information private from the bank. I then introduce a new form of payment which I call CBDC. Using this new form of payment does not timely reveal the entrepreneurs' production information to the bank. I show that introducing CBDC facilitates

[1] In IMF speech, *Winds of Change: The Case for New Digital Currency*, 2018

[2] European Central Bank (2020)

risk-sharing between entrepreneurs and the bank: Entrepreneurs with high privacy incentives would switch to CBDC, leaving the bank with less information about these borrowers and bearing more risk. The increased risk changes the bank's desired composition of assets between loans and liquid assets. In particular, a decrease in aggregate loan supply leads to a higher loan rate for anonymous borrowers and a lower bank return at the equilibrium. The more liquid portfolio lowers the probability of liquidity shortage thus to a *more stable* banking system. In terms of welfare, the bank depositors' welfare decreases and the entrepreneurs' welfare increases, even for those who value privacy less. What is more, CBDC brings in an externality that anonymous loan rate may adversely affect the informative loan rate. Without CBDC, a lower management cost leads to higher bank return and lower loan rate for borrowers. When CBDC is introduced, lowering management cost in CBDC payment leads to a higher loan rate for the informative payment borrowers. This is because the bank will shift the supply of funds in both types of payment until it is indifferent in lending. This suggests it may not be welfare-improving for the central bank, who has an advantage in managing CBDC payment flows, to provide information back to banks to lower management cost in anonymous payment. Mitigating information loss may hurt the informative payment users while improving the anonymous users and depositors' welfare.

The bank faces aggregate uncertainty in liquidity demand by depositors. Bank makes a portfolio choice that balances liquidity shortage and investment return in the spirit of Champ et al. (1996). What is more, I introduce private information and risky investment on the lending side. The borrowers (entrepreneurs) demand funds from bank to make production, and there are two means of payments to choose. They have private information about their production activities and the incentive to keep such information private: They generate non-transferable benefit if their investments are funded through without being interrupted by the bank. This feature is similar to Aghion and Bolton (1992) in which the borrowers value some important variables which cannot be verified and written in the contract. For example, if the production is terminated half-way it hurts the entrepreneur's reputation; or the entrepreneur values the experience of producing even though the project fails eventually. On the other hand, the competitive banks rely on the payment information to better monitor the loans. They can observe the return before investment matures if the borrowers use the debit card payment. They maximize depositors' utilities and will terminate the unprofitable projects to lower lending risks and avoid costly management. In the absence of anonymous payment, banks that have limited commitment can fully observe the loan returns and only continue to fund profitable loans, but at the cost of borrowers' privacy gains.

When the anonymous CBDC is introduced, borrowers can choose between the debit card and CBDC payment options in their production activities. The preferred choice depends critically on the loan rates of the two payment types and on the entrepreneur's private benefit. CBDC serves as a commitment device that allows the borrowers to preserve privacy gains. However, it causes information loss for the bank who cannot acquire the production information in time: the bank cannot observe the CBDC users' investment return before it matures. In addition, this may contribute to a higher management cost on banks. This management cost, which relies on the aggregate information available in the banking system, can be thought of detecting borrowers' misbehavior. This assumption captures the ECB's concern that anonymity may deteriorate bank assets. As a result, the CBDC payment users' loan rate will be higher. This result holds even when this management cost is the same for both types of payment. In equilibrium, only entrepreneurs with high privacy incentives will use CBDC, while others stick to the informative payment. The information loss from anonymous payment facilitates risk-sharing with additional management costs on the bank. It leads to a decrease in aggregate lending and a lower bank return at the equilibrium, in which both types of payments are active. The bank optimally chooses a more liquid investment portfolio and eventually leads to banks' less likely to experience liquidity shortages. Thus the bank is more stable despite being less informative about the loans in time. In addition, the debit card payment users' loan rate decreases as the returns from both types of payment users should be equal at the equilibrium. The entrepreneurs who do not value privacy much can also benefit from CBDC due to the decreased debit card loan rate compared to no CBDC case.

Lastly, I analyze the interaction between the loan rates in the anonymous and informative payment. I show that changes in the anonymous payment's loan rate can adversely affect the informative loan rate and provide implications for the potential role of the central bank. The CBDC payment's loan rate is higher due to more risk-sharing and additional management costs. The central bank issues digital currency and naturally has the advantage to document payment users' information. The central bank could direct information back to the banking system to lower the bank's management cost. However, with both credit and anonymity CBDC payments competing, it does not unambiguously improve welfare for all agents. The anonymous and informative payment users' loan rates interact, and they can move in the opposite direction when one of them changes. This is because both the supply and demand of funds in the two payments would adjust to clear the market.

Literature Review Many recent studies focus on privacy and the use of customers' data. For example, Garratt and Van Oordt (2021) study how the customers value privacy because they would face less price discrimination. Lee and Garratt (2021) show that payment competition leads to the data monopolist, while anonymity preserves the competitive market structure and improves customers' welfare. In another paper, Kang (2021) shows multiple equilibria exist when the seller can make profit from purchasing the consumers' private information and predicting their preferences. In all these papers, individuals favor anonymity for privacy concerns since they worry their information is used against their benefits, either by the transaction's counterparty or the third party. There is a similar concern in my model. If it acquires the information early, the bank values the entrepreneurs' private information and can take advantage of it. On the other hand, the entrepreneurs have privacy incentive that conflicts with the bank. This structure is similar to Aghion and Bolton (1992), Hart and Moore (1998), who discussed the incomplete contract between a penniless entrepreneur and a wealthy investor. In my model, whether the bank can or cannot acquire the information to stop funding unprofitable projects depends on the means of payment methods. This difference gives rise to the different loan rates in these two payment methods.

There is also a fast-growing literature on CBDC's potential impact on monetary policy and financial system [3]. For banking stability, there is concern for CBDC's disintermediation effect that it may crowd out bank deposits [4]. CBDC may also provide flight-to-safety and trigger bank runs as in Williamson (2021). Fernández-Villaverde et al. (2021) discuss the potential of having a central bank deposit monopolist, deterring bank runs. Monnet et al. (2021) consider an interest-bearing CBDC that competes with banks in lending activities. Competition from CBDC drives up the bank's monitoring effort, lowers asset riskiness, and improves welfare. Chiu et al. (2021) studied the effects of introducing a central bank digital currency when banks have market power. Keister and Monnet (2020) discussed the information perspective of CBDC on bank stability. In their work, the account-based CBDC provides additional information to the central bank so the bank runs can be detected and intervened early.

Comparing to the stability research above, I show anonymous CBDC's potential impact from information loss. The anonymous payment diverts information away from the bank and affects both bank's assets and liabilities. Bank's inability to kick out unprofitable projects leads to more risk-sharing between the borrower and lender, bringing in a high loan rate for anonymous payment borrowers and a lower bank return at the equilibrium. These affect bank's portfolio choice in the spirit of Champ et al. (1996) that the bank balances liquidity shortage probability and investment return. This provides a new aspect in a bank's portfolio choice: information loss contributes to choosing a more liquid portfolio when both the anonymous and informative payments exist.

The risk-sharing mechanism is also related to the 'Hirshleifer effect' as Hirshleifer (1971)

[3] See, for example, discussions by Bordo and Levin (2018), Meaning et al. (2018)

[4] For example, in the seminar works of Piazzesi and Schneider (2020), Bindseil (2020), Gross and Schiller (2021)

and the more recent work by Andolfatto et al. (2014). The later discusses the optimal information disclosure policy under a lack of policy commitment in the exchange economy. In my work, the lender has limited commitment and more risk-sharing emerges when the bank losses information. My focus is also different, which is on the information loss's impact on bank stability.

The rest of the paper is organized as follows. Section 2 describes the model environment, shows the borrower and bank's problems, and solves for the equilibrium without CBDC. Section 4 introduces CBDC payment, and the fraction of CBDC borrowers is endogenously determined. I compare the stability and welfare with and without anonymous CBDC. Section 5 discusses the central bank's potential role in mitigating banks' information loss. Section 6 concludes.

2 The Model

2.1 Entrepreneurs

There are three periods $t = 0, 1, 2$. At period 0, a continuum of penniless entrepreneurs with measure one needs funds to produce. All entrepreneurs consume at period 2 and have the same production technology. For one unit of good invested at $t = 0$, the investment generates R units with probability δ at $t = 2$; and 0 otherwise. An individual entrepreneur i chooses the production quantity q^i and consumption c_e^i, and production incurs a linear disutility $L(q^i) = -q^i$. In addition, an entrepreneur has private benefit b^i from production. In sum, an entrepreneur i has the following quasi-linear utility function:

$$u(c_e^i, q^i) = 2\sqrt{c_e^i} - q^i + I_{invest} \cdot b^i$$

Where b^i represents the non-transferable benefit which is non-transferable. The indicator $I_{invest} \in \{0, 1\}$ for this benefit. In particular, the entrepreneur benefits b^i only if the production activity is successfully funded through and matures at $t = 2$ (so $I_{invest} = 1$). If the production is terminated at $t = 1$, there will be no gains ($I_{invest} = 0$). Note that this private benefit provides the incentive for privacy, i.e., to keep the investment information private during production. For simplicity, I assume the private benefit is uniformly distributed across all borrowers as $b^i \sim U(0, \hat{b})$, with some upper limit $\hat{b}$.

The entrepreneur's production process involves transaction payments (i.e. payment flows like transactions for raw materials, etc.). The entrepreneur discovers his own production return privately at $t = 1$ based on information underneath these transactions. Importantly, if the lending bank can observe this payment information, it can also observe the production return at $t = 1$. The entrepreneur chooses a payment option that determines whether the banks can or cannot acquire the payment information during production. In particular, an entrepreneur can choose the informative "debit card" payment issued by the bank, enabling the bank to observe the payment activities, thus the investment's return at period 1. Alternatively, he can use the anonymous CBDC, so the bank cannot observe the return until the investment matures at period 2. Depending on his choice, the entrepreneur thus also faces two different loan rates: the anonymous CBDC loan rate (CBDC rate hereafter) and the informative debit card payment loan rate (debit card rate hereafter).

2.2 Depositors and the Representative Bank

At $t = 0$, there is also a continuum of depositors with measure one, each endowed with one unit of the single good and having the utility function $u(c) = ln(c)$. Among the depositors, a random fraction of π becomes impatient and needs consumption at $t = 1$, while the rest are patient and want to consume in period 2. The depositors pool their resources in the bank as in Diamond and Dybvig (1983) at $t = 0$, but the random variable π satisfying $\pi \sim U(0, 1)$ will be publicly realized at the beginning of $t = 1$.

A competitive bank collects funds from depositors and lends to entrepreneurs at $t = 0$. It can either hold the deposits as liquid cash reserves or lend to the entrepreneurs whose projects generate higher returns but only mature in period 2. Denote the return it offers to impatient and patient depositors as $c_1(\pi)$ and $c_2(\pi)$, respectively. The bank chooses the fraction γ of deposits to hold as a cash reserve and lends the rest $1-\gamma$ to entrepreneurs. When CBDC is introduced, the bank also chooses the fraction of lending θ, so that fraction $\theta(1-\gamma)$ lends to debit card entrepreneurs and $(1-\theta)(1-\gamma)$ to CBDC entrepreneurs. Importantly, I assume the bank has limited commitment in lending: If the bank observes negative present value projects, it will choose to terminate the loan. Thus for debit card borrowers, the bank is able to terminate the project if the bank realizes the investment is not profitable at period 1[5].

In addition, the bank incurs a management cost for its continuing loans at $t = 2$. I assume the informative payment's management cost holds as a constant $\bar{k}$. This cost is small relative to the investment return that $\bar{k} < min\{\frac{1}{2\delta}, \delta R - 1\}$. The anonymous payment's management cost is increasing in the fraction of borrowers using the anonymous payment:

$$k_{dc} = f(1 - I) \quad (1)$$

[5] For simplicity, I assume the means of payment is enforceable: if the entrepreneur borrows using the transparent bank credit payment, he cannot switch to CBDC after the loan is granted.

In which $I \in [0,1]$ represents the fraction of the informative payments in the economy, and it will be endogenously determined. It satisfies $f'(1-I) > 0$. It captures the idea that the bank's role in monitoring and asset management may be hampered by the available information in the banking system. Intuitively, the bank's ability to manage the projects decreases with less production information about the industry. Banks can easily know whether a project operates well by comparing it to other projects with more information available. The bank can detect the borrowers' misbehaving by analyzing and compare the payment flows among the entrepreneurs. On the other hand, less information (a smaller I) on the loans and the aggregate economy contributes to higher management costs. Note that due to this cost k, the bank would want to stop funding the investments that generate negative return investments (minus k). The entrepreneurs who use the debit card payment will not capture privacy gain when the investment fails.

2.3 Timeline

The following timeline summarizes all the descriptions above. At $t = 1$, the entrepreneurs request funds from the bank. The bank collects from depositors, chooses the fraction of lending amount, arranges payments for depositors. The loan rates depend on the payment methods the entrepreneur chooses in the production process. The debit card payment allows the bank to observe the realized return at $t = 1$ when the entrepreneur knows it. The CBDC prevents the bank from observing. At the beginning of period 2, management costs k incurs for any continuing loans. The entrepreneurs repay the loans, and lastly, the patient depositors withdraw.

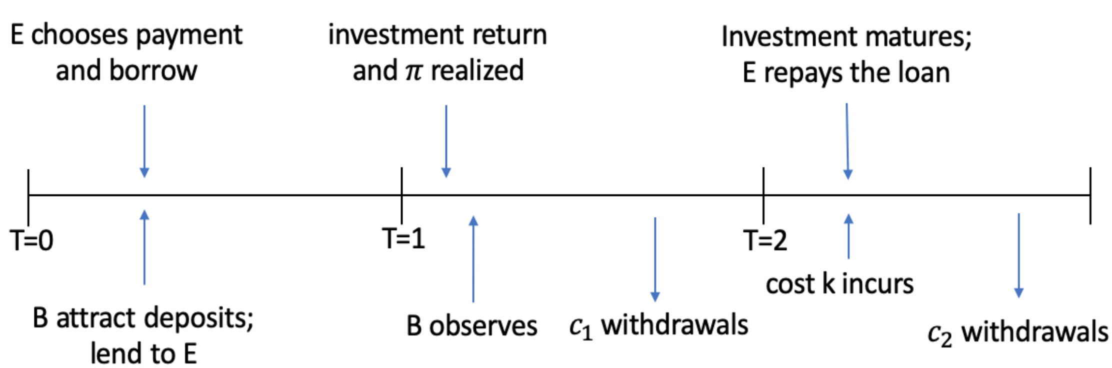

Figure 1. Timeline

3 Equilibrium Without CBDC

In this section, I solve for the equilibrium in which only the informative debit card payment is available to entrepreneurs. I will characterize the entrepreneur's problem for loan demand. Next, I solve for the bank's problem to derive the supply of loan. Then I will equate the supply and demand to find the equilibrium loan rate and discuss the bank stability characterized by the probability of liquidity shortage.

3.1 Entrepreneur's problem

At period 0, an entrepreneur i chooses his production level q_n^i when borrowing with the loan rate ρ_n. After borrowing from the bank, the entrepreneur will use the debit card payment to spend the funds. The debit card allows the lending bank to observe the borrowers' payment flows. In this sense, there is no information asymmetry as the bank can acquire the same information. Thus, the bank can also realize the investment return when it is known by the entrepreneur at period 1. The entrepreneur rationalizes this, knowing that the bank will terminate the unprofitable investments since the bank would choose not to pay the additional management cost k for investment with zero return. His project will be terminated with probability $1 - \delta$. So this entrepreneur i's problem is:

$$\max_{q_n^i} \quad \delta \cdot 2\sqrt{(R - \rho_n) \cdot q_n^i} - q_n^i + \delta \cdot b^i$$

The FOC gives his demand for loan as:

$$q_n^i = \delta^2 \cdot (R - \rho_n) \tag{2}$$

His demand depends on the given loan rate ρ_n, and the private benefit b^i does not affect his loan demand here. Note this equation also provides the aggregate demand for loan since there is one unit of entrepreneurs in total. In addition, the entrepreneur's indirect utility has two parts: the consumption from production after paying back the loan and the private benefit. The entrepreneur i's utility can be expressed as:

$$\Pi_n^i = \delta^2 \cdot (R - \rho_n) + \delta \cdot b^i \tag{3}$$

3.2 Bank's problem

At $t = 0$, a representative bank collects funds from depositors and lends to the entrepreneurs. The bank faces uncertainty in the demand for early withdrawals at period 1, so it maximizes the depositors' expected utilities by offering $c_{n1}(\pi)$ and $c_{n2}(\pi)$ for early and late depositors. In the competitive lending market, the representative bank takes the loan rate ρ_n as given. It chooses to hold a fraction γ_n of the deposit to prepare for $t = 1$ withdrawals, and lend the rest $(1 - \gamma_n)$ to the entrepreneurs. Since the uncertainty in aggregate liquidity demand only dissolves at $t = 1$, the bank may run out of its liquid assets at $t = 1$ when it meets the liquidity demand. In this paper, the bank is defined as unstable when it runs out of cash reserves, i.e. the bank experiences a liquidity crisis, as in Champ et al. (1996).

At period 1, the bank acquires the production information by observing the payment flows. The bank realizes the investment return as the entrepreneurs. The bank may terminate the loan since there is an small management cost $\bar{k}$ for each continuing loan: If the production return will be R at $t = 2$, the bank is willing to continue funding the investment; otherwise the bank will terminate the project that worth zero. Denote the bank's investment return as R_n:

$$R_n = \delta\rho_n - \delta\bar{k}$$

It shows the bank pools the loans together and only a fraction δ of the loans that generate R are worth continuing. The management cost paid by the bank is $\delta\bar{k}$. In sum, the bank's problem can be written in the following:

$$\begin{aligned}
\max_{\gamma_n, c_{n1}(\pi), c_{n2}(\pi), \alpha(\pi)} \quad & \int_0^1 [\pi \cdot lnc_{n1}(\pi) + (1-\pi) \cdot lnc_{n2}(\pi)] \cdot f(\pi) d\pi \\
\text{s.t.} \quad & \pi c_{n1}(\pi) = \alpha(\pi)\gamma_n \\
& (1-\pi)c_{n2}(\pi) \leq \gamma_n - \alpha(\pi)\gamma_n + (1-\gamma_n)(\delta\rho_n - \delta\bar{k}) \\
& 0 \leq \alpha(\pi) \leq 1
\end{aligned}$$

The choice $\alpha(\pi) \in [0, 1]$ represents the fraction of cash being used by the bank to pay the early depositors.

Whether the bank experiences liquidity shortage depends critically on the whether $\alpha = 1$ holds. In particular, it can be shown there is a threshold $\bar{\pi}_n$ such that if $\pi < \bar{\pi}_n$, the depositors' payoffs satisfy $c_{n1}(\pi) = c_{n2}(\pi) = \gamma_n + (1-\gamma_n)R_n$, and the bank has liquid assets $(\alpha < 1)$. For $\pi \geq \bar{\pi}_n$, the payoffs are $c_{n1}(\pi) = \frac{\gamma_n}{\pi}$ and $c_{n2}(\pi) = \frac{(1-\gamma_n)R_n}{1-\pi}$, while the bank runs out of cash reserves with $\alpha = 1$ and cannot treat all depositors the same[6]. Thus, the bank experiences a liquidity shortage thus is unstable when the realized $\pi > \bar{\pi}_n$.

[6] When $\pi = \bar{\pi}_n$, the bank chooses exactly $\alpha = 1$ with $c_1 = c_2$ and is defined as no liquidity shortage.

The choice of cash reserve can be solved from the FOC of γ_n:

$$1 - \gamma_n = \int_{\bar{\pi}_n}^{1} F(\pi) d\pi \tag{4}$$

where $\bar{\pi}_n = \frac{1}{1+(\frac{1}{\gamma_n}-1)(\delta\rho_n - \delta\bar{k})}$, and the term $F(.)$ is the cumulative distribution of π. The investment return R_n is greater than one because $\bar{k}$ is relatively small with $\delta R - \bar{k} > 1$. Lastly, note that this equation (4) also implicitly provides the aggregate supply of funds, $1 - \gamma_n$, as a function of the loan rate ρ_n.

3.3 Equilibrium and Bank Liquidity

Denote the equilibrium cash reserve ratio as γ_n^*, and the equilibrium loan rate as ρ_n^*. I apply the uniform distribution on π and can rewrite (4) as:

$$1 - \gamma_n = \frac{1}{2} - \frac{1}{2}(\frac{1}{1 + (\frac{1}{\gamma_n} - 1)(\delta\rho_n - \delta\bar{k}})^2 \tag{5}$$

I equate the supply and demand for loans to get the market clearing condition:

$$1 - \gamma_n = \delta^2 (R - \rho_n) \tag{6}$$

Condition (5) and (6) pin down the equilibrium with $\{\gamma_n^*, \rho_n^*\}$. Note that the individual entrepreneur's privacy gain does not affect the equilibrium loan rate while the bank has limited commitment to terminate the loans. Rewrite (6) as $\delta\rho_n = \delta R - \frac{1}{\delta}(1-\gamma_n)$ and replace $\delta\rho_n$ in (4), now I can get the condition that pins down γ_n^*:

$$1 - \gamma_n^* = \frac{1}{2} - \frac{1}{2}(\frac{1}{1 + (\frac{1}{\gamma_n^*} - 1)(\delta R - \frac{1}{\delta}(1 - \gamma_n^*) - H_n)})^2 \tag{7}$$

In which the term $H_n = \delta\bar{k}$, capturing the management cost incurs. It can be shown that the RHS of condition (7) is concave and decreasing in γ_n. If $\gamma = 1$, the RHS equals 0 with the slope smaller than minus one[7]. Thus, there is a unique interior γ_n^* that solves the equation, and there is a unique interior solution for ρ_n^*. Figure 2 below graphically shows the optimal γ_n^* at the equilibrium. The lines g_0 and g_1 represent the LHS and RHS of (7). Since $0 \le \gamma^* \le 1$, the interior intersection point is the only solution γ_n^*.

[7] Which does not depend on the assumption on the distribution of π.

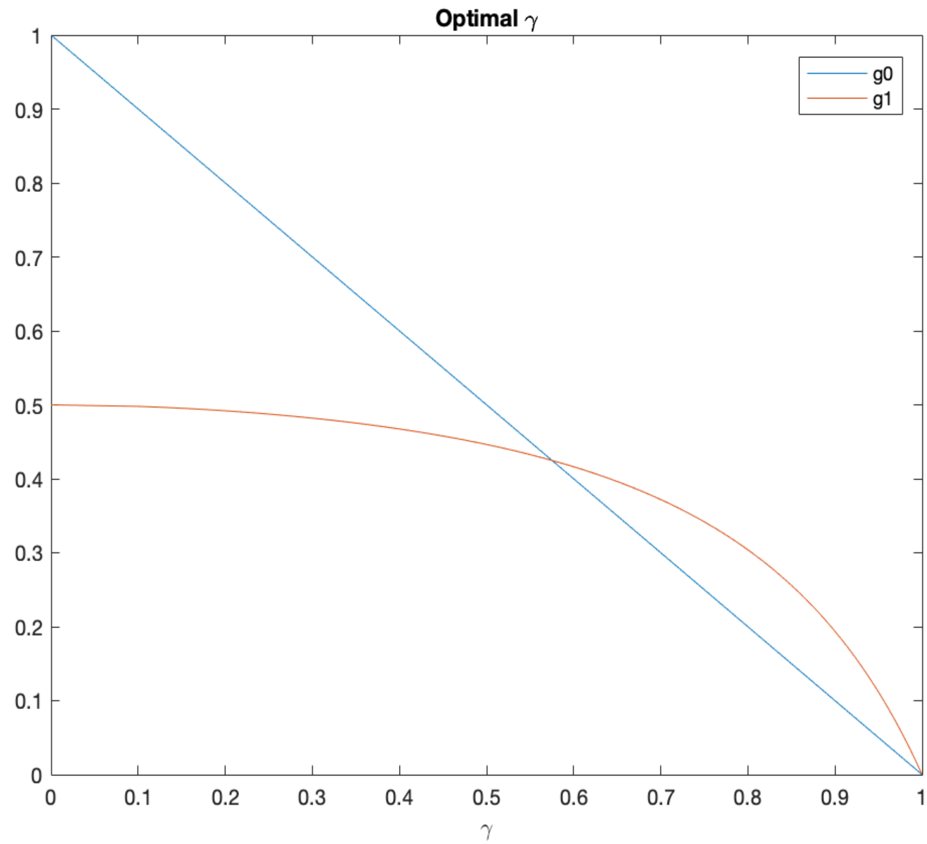

Figure 2. Optimal γ_n

Recall that if the realized liquidity demand π is larger than $\bar{\pi}_n$, then the bank is running out of cash reserves and is said to experience the liquidity shortage. The marginal $\bar{\pi}_n^*$ at the equilibrium naturally measures the probability of running out of cash reserve at $t = 1$, which is determined by the γ_n^*. A more liquid investment portfolio leads to a lower $\bar{\pi}_n$ and a more stable bank.

In addition, if the bank's loan becomes more costly to manage, the bank will optimally choose to hold a more liquid portfolio with higher cash reserves γ_n^*. This can be seen from the bank's return in (7), which is expressed as $R_n = \delta R - \frac{1}{\delta}(1 - \gamma_n) - \delta\bar{k}$. For some $k' > \bar{k}$, bank's return R_n would decrease. The lower R_n would further contribute to a more liquid portfolio choice with higher cash reserve γ_n^* by the bank.

Proposition 1. *There is an unique interior equilibrium with $\{\gamma_n^*, \rho_n^*\}$ in the economy. If $\bar{k}$ increases, γ_n^* and $\bar{\pi}_n$ increase with a more liquid bank portfolio.*

In the absense of CBDC, the competitive banking system has a unique equilibrium in which the bank makes a portfolio choice to balances the liquidity shortage and higher investment return. The proposition also shows the mechanism of how the changed management cost can affect bank return and bank stability: if the management cost increases, the bank optimally chooses a more liquid portfolio because the investment is less attractive. Thus, the liquidity shortage threshold $\bar{\pi}_n$ is also higher. In the next section, I will show the introducing CBDC contributes to information loss by the bank. The anonymous payment will enforce more risk-sharing between the entrepreneurs and the bank. The bank's management cost and return change due to the risk-sharing, as well as its liquidity choice and stability.

4 Equilibrium With CBDC

Now I introduce the anonymous CBDC payment that can substitute the informative debit card payment. The anonymous payment method serves as a commitment tool that allows the entrepreneurs to keep the realized return private at $t = 1$. The bank can no longer timely acquire the investment's information. The bank's ability to manage the loans is affected, reflecting from more risk-sharing and a different CBDC payment management cost k_{dc}. I will introduce the entrepreneur's problem and bank's problem respectively, find the equilibrium that these two payments coexists and make comparison.

4.1 Entrepreneur's Problem

The entrepreneur can now choose to use the anonymous CBDC payment under some loan rate ρ_0. Now the difference is that if he chooses CBDC, the entrepreneur i ensures his private benefit b^i. The bank cannot realize the production's return since it cannot analyze the payment flows of the entrepreneurs. The entrepreneur i's problem can be written as:

$$\max_{q_0^i} \quad \delta \cdot 2\sqrt{(R-\rho_0)\cdot q_0^i} - q_0^i + b^i \cdot$$

Compared with no CBDC case, the anonymous payment ensures the entrepreneur's private gain b^i. The entrepreneur's demand for loan is:

$$q_0^i = \delta^2 \cdot (R - \rho_0) \tag{8}$$

His indirect utility as the function of the CBDC loan rate is:

$$\Pi_0^i = \delta^2 \cdot (R - \rho_0) + b^i \tag{9}$$

Now I denote the debit card rate as ρ_1 when CBDC payment is introduced. Similar to equations (2) and (3), if the entrepreneur chooses the credit payment, his demand for loan and utility can be expressed as:

$$q_1^i = \delta^2 \cdot (R - \rho_1) \tag{10}$$

$$\Pi_1^i = \delta^2 \cdot (R - \rho_1) + \delta b^i \tag{11}$$

Importantly, an entrepreneur's payment choice depends on his utility Π_0^i and Π_1^i, which are affected by the utility gain b^i. For given loan rates, there is a threshold $\bar{b}$ such that if $b^i > \bar{b}$, then $\Pi_0^i > \Pi_1^i$. This threshold $\bar{b}$ satisfies $\Pi_0^i = \Pi_1^i$ and can be written as:

$$\delta^2 \cdot (\rho_0 - \rho_1) = (1 - \delta) \cdot \bar{b} \tag{12}$$

This condition shows that those entrepreneurs who have high private gains with $b^i > \bar{b}$ will use anonymous CBDC, while those with $b^i < \bar{b}$ will use the transparent credit payment. This $\bar{b}$ will be endogenously determined in the equilibrium when the loan rates are pinned down.

4.2 Bank's problem

The representative bank now serves the entrepreneurs using either types of payment. The bank provides two loan rates, the CBDC rate ρ_0 and debit card rate ρ_1, depending on the payment method will be used by the entrepreneur after receiving the funds. The management cost for CBDC payment user is k_{dc}, which can be different from $\bar{k}$. For simplicity, I assume an explicit form of this management cost k_{dc} of CBDC payment is:

$$k_{dc} = \bar{k} + \beta \cdot (1 - \bar{b}) \tag{13}$$

In which β is a parameter that satisfies $\beta + \bar{k} \leq 1$. By assumption, $k_{dc} \geq \bar{k}$. This is to capture the idea that anonymous payment may negatively affect the bank's ability to monitor assets as concerned by the ECB. This assumption is general, and it will be shown that if $k_{dc} = \bar{k}$ the main mechanism in this paper holds.

In the absence of CBDC, all entrepreneurs use the payment offered by commercial banks, so $I = 1$ and $k = \bar{k}$. With CBDC, the entrepreneurs' information are diverted away. Now the fraction of information available is $I = \bar{b}$, since only the debit card users' payment information can be observed. Note the cost k_{dc} is endogenously determined by the fraction of CBDC users at the equilibrium when $\bar{b}$ is pinned down.

For any given k_{dc}, the bank's return from this anonymous payment R_{b0} can be written as:

$$R_{b0} = \delta\rho_0 - k_{dc}$$

In which $k_{dc} = \bar{k} + a(1 - I)$, a function of loan information available to the banks. Let R_{b1} be the debit card payment return:

$$R_{b1} = \delta\rho_1 - \delta\bar{k}$$

Note the CBDC management cost is not multiplied by δ, since the bank cannot kick out the unprofitable loans at $t = 1$ and will always incur this cost. In this sense, more risk-sharing incurs between the entrepreneurs and the bank.

The bank also needs to choose the fraction of loan supply to the CBDC payment entrepreneurs. It keeps a fraction θ of the lending to the debit card payment, so that $\theta(1-\gamma)$ lends through the debit card payment and the remaining $(1-\theta)(1-\gamma)$ through the CBDC payment. In sum, the bank's problem is:

$$\begin{aligned}
\max_{\gamma,\theta,c_1(\pi),c_2(\pi),\alpha(\pi)} \quad & \int_0^1 [\pi \cdot lnc_1(\pi) + (1-\pi) \cdot lnc_2(\pi)] \cdot f(\pi) d\pi \\
\text{s.t.} \quad & \pi c_1(\pi) = \alpha(\pi)\gamma \\
& (1-\pi)c_2(\pi) \leq \gamma - \alpha(\pi)\gamma + (1-\gamma)[\theta R_{b1} + (1-\theta)R_{b0}] \\
& 0 \leq \alpha(\pi) \leq 1 \\
& 0 \leq \theta \leq 1
\end{aligned}$$

The choice $\alpha(\pi) \in [0, 1]$ represents the fraction of cash reserve used to pay early depositors. Similarly, there is a threshold $\bar{\pi}$ that for $\pi > \bar{\pi}$ the bank runs out of cash reserve. The FOC with respect to γ can be written as:

$$1 - \gamma = \int_{\bar{\pi}}^1 F(\pi) d\pi \tag{14}$$

where $\bar{\pi} = \frac{1}{1+(\frac{1}{\gamma}-1)R_b}$, and $R_b = \theta R_{b1} + (1-\theta)R_{b0}$.

It can be seen that the solution of γ needs to satisfy $0 < \gamma < 1$, otherwise either early or late depositors get zero payoff and negative infinity utility. In addition, both types of payment are active with $0 < \theta < 1$, otherwise the loan rate for either type of lending would be as high as R for any positive demand in that type. This can be seen from the market clearing conditions below:

$$\theta(1-\gamma) = \int_0^{\bar{b}} \delta^2(R-\rho_1)\cdot f(b^i)db^i \tag{15}$$

$$(1-\theta)(1-\gamma) = \int_{\bar{b}}^1 [\delta^2(R-\rho_0)]\cdot f(b^i)db^i \tag{16}$$

The first equation is the clearing condition for debit card payment users, and the second is the clearing condition for anonymous CBDC payment users. Thus, the bank would be indifference between the two types of lending with the interior θ^* that $R_{b0} = R_{b1}$. In fact, these properties allow me to simplify the conditions to pin down the equilibrium loan rates ρ_0^*, ρ_1^* and γ^*.

4.3 Equilibrium and Bank Liquidity

For given loan rates ρ_1 and ρ_0, when the bank is indifference between the two lending payments, it should satisfy:

$$1-\gamma = \int_{\frac{1}{1+(\frac{1}{\gamma}-1)(\delta\rho_1-\delta\bar{k}))}}^1 F(\pi)d\pi \tag{17}$$

$$1-\gamma = \int_{\frac{1}{1+(\frac{1}{\gamma}-1)(\delta\rho_0-k_{dc})}}^1 F(\pi)d\pi \tag{18}$$

Note the second condition can be replaced by $R_{b1} = R_{b0}$, so:

$$\delta(\rho_0-\rho_1) = (1-\delta)\bar{k} + \beta(1-\bar{b}) \tag{19}$$

I rewrite the condition (12) below, which shows the marginal entrepreneur is indifference to use either type of payment:

$$\delta(\rho_0-\rho_1) = \frac{1-\delta}{\delta}\cdot\bar{b}$$

These conditions provide the equilibrium solutions for $\{\gamma^*, \rho_0^*, \rho_1^*, \bar{b}^*, k_{dc}^*, \theta^*\}$.

Now I investigate whether there is an interior equilibrium under these conditions with both payments being active. To begin with, combining (12) and (19) we can see that at the equilibrium:

$$\frac{1-\delta}{\delta}\bar{b}^* = (1-\delta)\bar{k} + \beta(1-\bar{b}^*) \tag{20}$$

Then the equilibrium $\bar{b}^*$ can be solved from (20) as:

$$\bar{b}^* = \frac{\beta + (1-\delta)\bar{k}}{\beta + (\frac{1}{\delta} - 1)} \tag{21}$$

Under the assumption that $\bar{k}$ is small (smaller than 1), it can be seen that $\frac{1}{\delta} - 1 > (1-\delta)\bar{k}$, and the equilibrium $\bar{b}^* < \hat{b} = 1$. What is more, as the marginal information loss β is higher, the equilibrium $\bar{b}^*$ will be higher so a larger fraction of borrowers will stick to use the debit card payment.

Lemma 1. *As marginal information loss β is higher, a larger fraction of borrowers will stay using debit card payment. The equilibrium management cost k^*_{dc} increases.*

The positive $\bar{b}^*$ leads to $\bar{k} < k^*_{dc} < 1$. The equilibrium monitoring cost is higher for anonymous payment because anonymity leads to information loss on the banks' side with additional cost. When the marginal information loss is higher (bigger β), the equilibrium $\bar{b}^*$ corresponds to higher equilibrium debit card users and it is costly to manage the anonymous payment users' loans.

The equilibrium conditions can be further simplified. Adding up (15) and (16) to drop θ, I can further simplify the equilibrium conditions to:

$$\delta\rho_0^* = \delta\rho_1^* + \frac{(1-\delta)}{\delta}\bar{b}^* \tag{22}$$

$$1-\gamma^* = \frac{1}{2} - \frac{1}{2}\left(\frac{1}{1 + (\frac{1}{\gamma^*} - 1)(\delta\rho_1^* - \delta\bar{k})}\right)^2 \tag{23}$$

$$1-\gamma^* = \int_0^{\bar{b}^*} \delta^2(R-\rho_1^*)\cdot f(b^i)db^i + \int_{\bar{b}^*}^1 \delta^2(R-\rho_0^*)f(b^i)db^i \tag{24}$$

The first condition is from (14). The second condition comes from (15) and applies the uniform distribution on π. The third condition adds up (12) and (13) that equalizes the supply and demand for loans. I can use (22) to replace $\delta\rho_0^*$ and rewrite (24) as:

$$\begin{aligned}
1-\gamma^* &= \int_0^{\bar{b}^*} [\delta^2(R-\rho_1^*)]\cdot f(b^i)db^i + \int_{\bar{b}^*}^1 [\delta^2(R-(\rho_1^* + \frac{(1-\delta)}{\delta^2}\bar{b}^*)]f(b^i)db^i \\
&= \delta^2(R-\rho_1^*) - (1-\delta)\bar{b}^*\cdot\int_{\bar{b}^*}^1 f(b^i)db^i \\
&= \delta^2(R-\rho_1^*) - (1-\delta)\bar{b}^*[1-F(\bar{b}^*)]
\end{aligned}$$

Where $F(\bar{b}^*)$ is the cumulative of entrepreneurs using debit card payment. $F(\bar{b}^*) = \bar{b}^*$

because of the uniform assumption over b^i. I rewrite $\delta\rho_1^*$ as:

$$\delta\rho_1^* = \delta R - \frac{(1-\gamma^*)}{\delta} - \frac{(1-\delta)}{\delta}\bar{b}^*[1-\bar{b}^*]$$

Then replace $\delta\rho_1$ in (23):

$$1-\gamma^* = \frac{1}{2} - \frac{1}{2}\left(\frac{1}{1+(\frac{1}{\gamma^*}-1)(\delta R - \frac{1}{\delta}(1-\gamma^*) - H_{coe}(\bar{b}^*))}\right)^2 \tag{25}$$

In which

$$H_{coe}(\bar{b}^*) \equiv \frac{(1-\delta)}{\delta}\bar{b}^*[1-\bar{b}^*] + \delta\bar{k} \tag{26}$$

Note this term does not depend on γ and loan rates. The RHS of (25) is a concave function of γ with slope smaller than minus one at $\gamma = 1$, which ensures a unique interior γ^*.

Proposition 2. *There is an unique interior equilibrium with* $\{\gamma^*, \rho_0^*, \rho_1^*, \bar{b}^*, \theta^*\}$ *that the anonymous CBDC and debit card payments coexist.*

The anonymous payment substitutes the debit card payment and diverts funds to the less informative borrowing. The relative size of monitoring cost k_{dc} to firms' non-transferable gain b^i determines the fraction of lending in each type. The proposition shows at the equilibrium, both types of lending are active. The loan rates are endogenously determined and one type of payment cannot completely drive out the other one due to the changing loan rates.

4.4 Comparison: Stability and Welfare

Now I can compare the stability and welfare with and without CBDC cases. If we compare condition (25) with no CBDC case's condition (7), the difference lies between the terms H_{credit} and H_{coe}. It can be shown:

$$\Delta H = H_{credit} - H_{coe} = (k_{dc}^* - \delta\bar{k}) \cdot [1-\bar{b}^*] \tag{27}$$

So that $H_{credit} > H_{coe}$, and the difference captures the additional risk-sharing between the bank and entrepreneurs. It represents the fraction of anonymity borrowers, $1-\bar{b}^*$, each incur the additional management cost $k_{dc}^* - \delta\bar{k}$ on the bank because the bank cannot terminate unprofitable projects. Denote the RHS of (25) as g_2, the following graph compares the optimal choice of γ with and without CBDC.

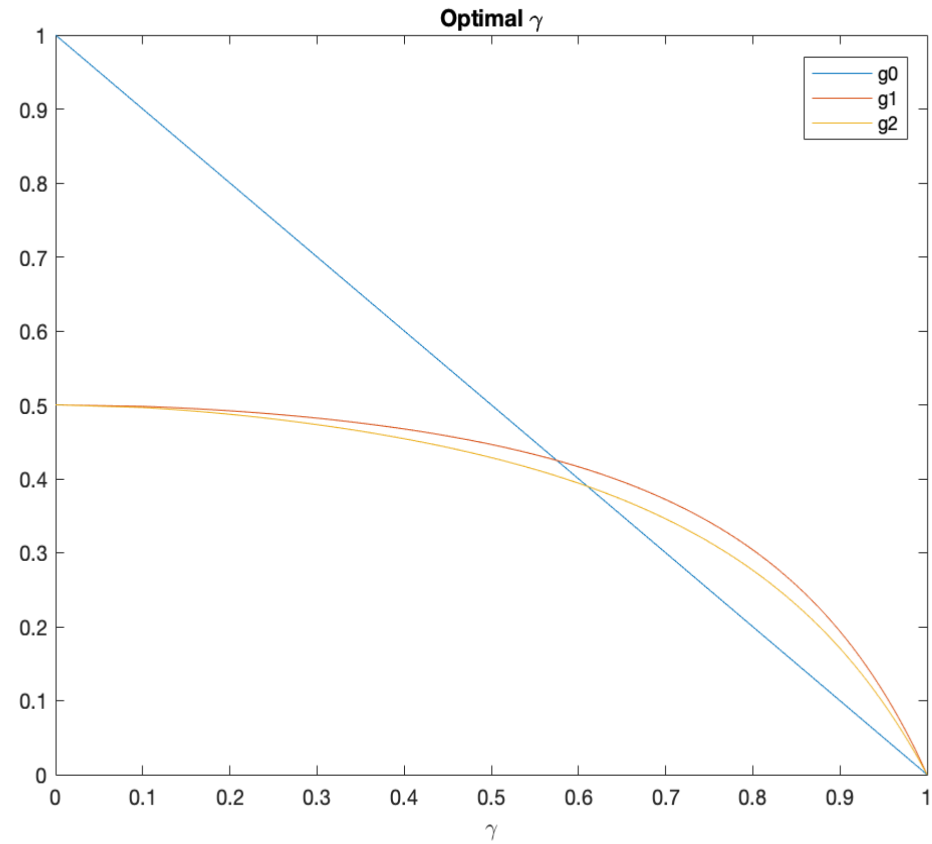

Figure 3. Optimal γ with and without CBDC

The intersection points of g_0 and g_2 is the optimal solution of γ^*, comparing to without CBDC's intersection point of g_0 and g_1. The more risk-sharing with additional cost makes the lending more costly. The bank's investment return decreases at the equilibrium when the less informative means of payment (CBDC) can substitute the traditional informative one (debit card). Without CBDC, the loans are less costly and the bank chooses a less liquid investment portfolio, which contributes to a lower γ^*_{credit}. Thus the reserve threshold $\bar{\pi}$ is lower in the equilibrium, which corresponds to the more likelihood to experience the liquidity shortage at $t = 1$. With CBDC, the lower RHS g_2 relative to g_1 corresponds to the risk-sharing and lower investment return. The bank optimally chooses a more liquid portfolio with a larger γ^*, leading to a higher liquidity threshold $\bar{\pi}$. In this sense, the anonymous CBDC indirectly promotes bank stability since it enforces a more liquid portfolio when there is more risk-sharing sourced from introducing the less informative means of payment.

Proposition 3. *Compared with no CBDC case, the bank is more stable with $\gamma^* > \gamma^*_n$ and $\bar{\pi} > \bar{\pi}_n$.*

There are two factors that lowers the bank's investment return. The first is the risk-sharing that CBDC users brings in. Second, bank's ability to manage its loans is hampered. This is captured by $k^* > k_0$ at the equilibrium, reflecting the point that monitoring the less informative assets is harder. In fact, the main mechanism still holds if I turn off this second mechanism by imposing $\beta = 0$, so that managing the less informative loans does not incur additional management cost and $k_{dc} = \bar{k}$. In this sense, the condition (27) can be further reduced to $(1-\delta)\bar{k}(1-\bar{b}^*)$, which states more clearly that with probability $(1-\delta)$, the $(1-\bar{b}^*)$ CBDC entrepreneurs' investment that generates zero return is funded through at the additional cost $\bar{k}$. The more risk-sharing still contributes to more liquid portfolio and the more stable bank. In this sense, the result is robust even when anonymous payment does not hamper bank's ability to management the loans.

In terms of welfare, the introduction of CBDC payment contributes to higher welfare for entrepreneurs. The equilibrium debit card rate is lower comparing to no CBDC case, $\rho_1^* < \rho_n^*$, and the debit card payment users can benefit from such lower loan rate. This result comes from the fact that at the equilibrium, the aggregate demand for loans decrease due to more risk-sharing with additional management cost. The bank's return is lower from entrepreneurs using CBDC payment, while the market clearing condition enforces the return on both types of lending holds equal. Since the management cost $\bar{k}$ holds for loans under the debit card payment, the equilibrium ρ_1^* has to be lower. As a result, the entrepreneurs using debit card payment benefit from the lower loan rate even though they do not value privacy. Meanwhile, the CBDC payment users are also better off because they are willing to accept a higher loan rate ρ_0^* to preserve privacy and shift to the less informative CBDC payment. Thus, both types of entrepreneurs are better off and the bank depositors are worse off since the lower investment return.

Proposition 4. *The equilibrium loan rate satisfies $\rho_1^* < \rho_n^*$. Compared with no CBDC case, entrepreneurs using both types of payment are better off. $W_e > W_{ne}$ and $W_d < W_{nd}$.*

In sum, introducing CBDC payment favors the entrepreneurs and lowers the welfare of depositors. The equilibrium bank return $R_b < R_n$ due to the additional risk-sharing between entrepreneurs and the bank. Introducing anonymity CBDC brings in this welfare tradeoff between the lender (depositors) and the borrowers (entrepreneurs), thus the impact on aggregate welfare in the economy is ambiguous.

5 Information and Loan Rates

The anonymous CBDC payment that potentially substitutes the informative payment can lead to more risk sharing between borrowers and the lender. The information loss may also deteriorate bank's ability to manage the loans, reflecting from k_{dc}. Meanwhile, the CBDC is issued and managed by the central bank, who serves public goals and has no incentive for making profit. The central bank naturally has an advantage in managing the payment information within the digital currency payment system. This brings in the discussion of whether and how the payment information should be managed. The implication relies on the interaction between the loan rates ρ_0^* and ρ_1^*, which can respond differently to the information loss from the anonymous payment. Managing the CBDC payment information to lower the bank's management cost may not necessarily be welfare-improving.

To begin with, consider the situation that the central bank reveals all the payment information to the bank. Then the bank could fully acquire entrepreneurs payment information again and observe the investment return. It would switch to the no CBDC case by figuring out which investment can be terminated at $t = 1$. There will be no risk sharing again between the borrowers and banks. In this sense, providing all the information to the bank, especially allowing banks to observe the investment return early, reveals too much information.

The alternative way for the central bank is to affect k_{dc}, which helps lower the management cost solely. In this paper's context, suppose the central bank can affect the marginal information cost of the bank. This is achieved by allowing the central bank to have a choice variable $\beta_p \in [0, 1]$ to replace the coefficient β:

$$k_{dc} = \bar{k} + \beta_p(1 - \bar{b})$$

By choosing β_p, the central bank can affect the equilibrium k_{dc} and the fraction of CBDC borrowers. Following Lemma 1, the central bank could lower the cost k_{dc} by choosing a lower β. It can be seen if $\beta_p = 0$, the bank bears no additional information loss from CBDC users, i.e. $k_{dc}^* = \bar{k}$, the model is simplified and the CBDC payment becomes only a commitment devise to ensure borrowers to receive privacy gains. If $\beta_p = 1$, the central bank facilitates risk-sharing and borrowers capture privacy gain more easily. However, with both payments coexist, the changing β_p and k_{dc} may adversely affect the informative payment users. The following proposition provides comparative analysis on the loan rates change caused by the marginal information cost β_p.

Proposition 5. *As β_p decreases, k_{dc}^* and $\bar{b}^*$ both decrease. The CBDC rate ρ_0^* unambiguously decreases. The debit card rate ρ_1^* decreases if $\beta_p < \bar{A}$; and increases otherwise.*

The term $\bar{A}$ is a constant that $\bar{A} = (1 - 2\delta\bar{k})(\frac{1}{\delta} - 1)$. When β_p decreases, the per loan management cost k_{dc}^* decreases and the CBDC payment is more attractive. The fraction of debit card payment users, $\bar{b}^*$ becomes smaller. It is less costly to lend to a CBDC entrepreneur and the CBDC rate ρ_0^* decreases. Meanwhile, the equilibrium bank return changes, depending on the change from the aggregate cost in lending captured by $\Delta H = (k^*_{dc} - \bar{k}) \cdot (1 - \bar{b}^*)$ as condition (27). The debit card rate ρ^*_1 is adversely affected by it: if the aggregate cost ΔH increases, bank's loan return decreases. Since the management cost on debit card payment $\bar{k}$ holding constant, the equilibrium ρ^*_1 has to be lower when market clears and the returns from the two types of lending hold equal.

In addition, recall the ΔH has the extensive-intensive tradeoff: when β_p decreases, the per loan cost k^*_{dc} decreases and the amount of CBDC users increases. Thus, the changing β_p can have ambiguous effect on the debit card rate ρ^*_1. Intuitively, if β_p is sufficiently small with a low k^*_{dc}, then the aggregate management cost $H_{\bar{b}^*}$ also decreases even though many entrepreneurs choose CBDC payment. The debit card payment rate ρ^*_1 will thus increase, the fund supply for debit card payment users decreases relative to the supply for CBDC users when market clears. On the other hand, if β_p is relatively high, a decrease in β_p lowers the per loan k_{dc}. But with more CBDC users, the aggregate cost ΔH increases. This contributes to lower equilibrium return and the ρ^*_1 is lower when market clears. The following figure shows the pattern of ρ^*_0 and ρ^*_1 and their relation with the amount of informative payment users $\bar{b}^*$ at the equilibrium.

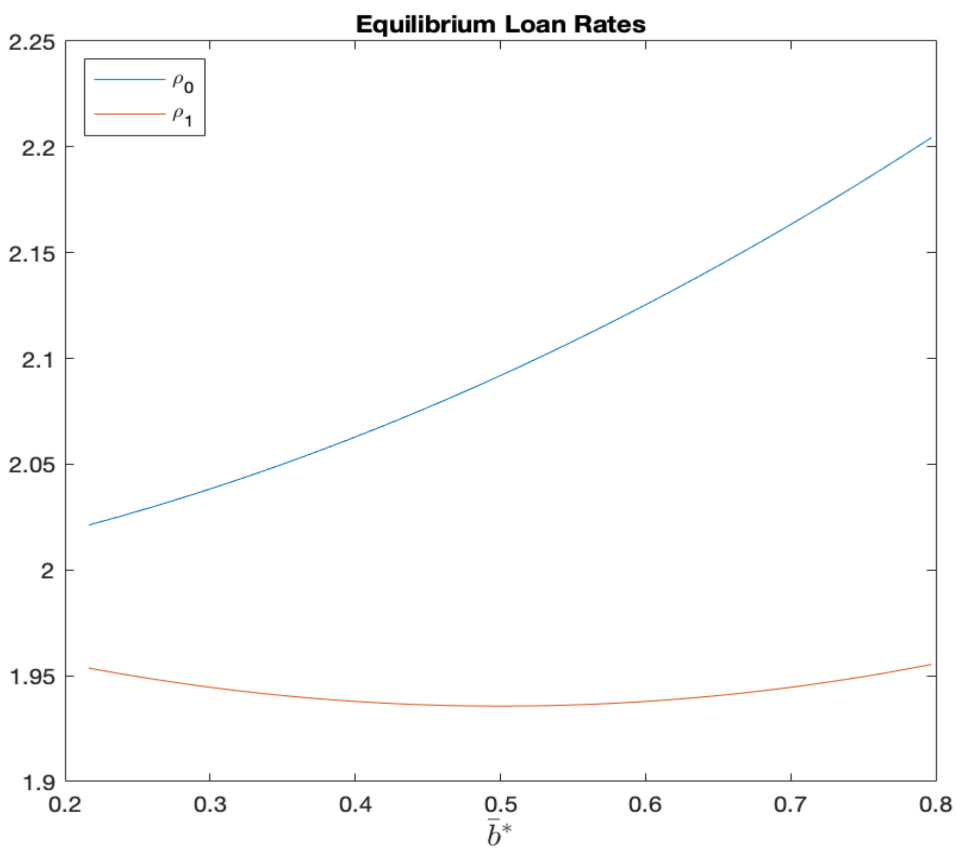

Figure 4. Equilibrium Loan Rates

It shows the equilibrium rates ρ_0^* and ρ_1^* can move in the opposite direction, depending on the amount of informative payment users $\bar{b}^*$ or equivalently the k_{dc}^* as β_p changes. In this example, the cutoff $\bar{b}^* = \frac{1}{2}$. As β_p decreases, the aggregate cost ΔH will first increase and then decrease due to the extensive and intensive margin tradeoff that always holds. The impact on the informative payment's loan rate ρ_1 is thus non-monotone as well.

This result suggests a key externality from managing the CBDC payment. When there is no CBDC, a decrease in the bank's management cost $\bar{k}$ contributes to higher bank return and lower loan rate. The gains from lowering cost always benefit both the bank depositors and the entrepreneurs. With CBDC payment, a decrease in CBDC cost k_{dc} may lead to a higher loan rate ρ_1 that could hurt the informative payment users. The entrepreneurs using informative payment can be adversely affected through the rising loan rate. The central bank may need to balance the welfare not only between the lender and borrowers, but also between the two types of payment users.

6 Conclusion

In this paper, I study the potential macroeconomic implications of the anonymous CBDC payment on bank stability and welfare in the lending market. The anonymous CBDC payment can preserve entrepreneurs' privacy while substituting the traditional informative payment. The information available for the bank is relatively limited comparing to the case that the bank can better monitor and manage the loans, when the entrepreneurs can only have the exclusive arrangement with the bank. The anonymous CBDC payment serves as a commitment devise and allows borrowers to fully capture their non-pledgeble gains. It enforces more risk sharing between the bank and entrepreneurs, leading to lower bank return, less aggregate lending with the introduction of a higher anonymous loan rate. These results hold even when the information loss does not hamper bank's ability to monitor the loans. The bank that balances liquidity shortage and loan return will optimally respond by choosing a more liquid portfolio, which eventually contributes to a more stable bank. This result is contrary to the concern that the loss of information would increase banks' risks and make them unstable.

Introducing anonymous payment favors the entrepreneurs while decrease the welfare of the bank depositors. Interestingly, the entrepreneurs who value less about their privacy can also benefit from introducing CBDC although they stay using the informative payment. This is because the loan rate of the informative payment method becomes lower as the aggregate demand decreases. The anonymous and informative loan rates are inter-connected and brings in a new externality. With CBDC, managing the anonymous payment information can negatively affect the other type of payment users' welfare, while such effect does not exist when there is only one type of payment to manage. In this sense, the central bank should be cautious in managing the payment information that flows within CBDC payment system.

References

Aghion, Philippe and Patrick Bolton (1992) "An incomplete contracts approach to financial contracting," *The review of economic Studies*, 59 (3), 473–494.

Andolfatto, David, Aleksander Berentsen, and Christopher Waller (2014) "Optimal disclosure policy and undue diligence," *Journal of Economic Theory*, 149, 128–152.

Berg, Tobias, Valentin Burg, Ana Gombović, and Manju Puri (2020) "On the rise of fintechs: Credit scoring using digital footprints," *The Review of Financial Studies*, 33 (7), 2845–2897.

Bindseil, Ulrich (2020) "Tiered CBDC and the Financial System," *ECB Working Paper*, https://ssrn.com/abstract=3513422.

Bordo, Michael and Andrew Levin (2018) "Central bank digital currency and the future of monetary policy," in M.D.Bordo, J.H.Cochrane, and A. Seru ed. *The Structural Foundations of Monetary Policy*: Hoover Institution Press Publication 687.

Champ, Bruce, Bruce D Smith, and Stephen D Williamson (1996) "Currency elasticity and banking panics: Theory and evidence," *Canadian Journal of Economics*, 828–864.

Chiu, Jonathan, Mohammad Davoodalhosseini, Janet Hua Jiang, and Yu Zhu (2021) "Bank market power and central bank digital currency: Theory and quantitative assessment," Bank of Canada Staff Working Paper 2019-20, revised March.

Diamond, Douglas W and Philip H Dybvig (1983) "Bank runs, deposit insurance, and liquidity," *Journal of political economy*, 91 (3), 401–419.

European Central Bank (2020) "Report on a digital euro," October, www.ecb.europa.eu/euro/html/digitaleuro-report.en.html.

Fernández-Villaverde, Jesús, Daniel Sanches, Linda Schilling, and Harald Uhlig (2021) "Central bank digital currency: Central banking for all?" *Review of Economic Dynamics*, 41, 225–242, https://doi.org/10.1016/j.red.2020.12.004, Special Issue in Memory of Alejandro Justiniano.

Garratt, Rodney J and Maarten RC Van Oordt (2021) "Privacy as a public good: a case for electronic cash," *Journal of Political Economy*, 129 (7).

Gross, Jonas and Jonathan Schiller (2021) "A Model for Central Bank Digital Currencies: Implications for Bank Funding and Monetary Policy," https://ssrn.com/abstract=3721965.

Hart, Oliver and John Moore (1998) "Default and renegotiation: A dynamic model of debt," *The Quarterly Journal of Economics*, 113 (1), 1–41.

Hirshleifer, Jack (1971) "The private and social value of information and the reward to inventive activity," in *Uncertainty in economics*, 541–556: Elsevier.

Kang, Kee-Youn (2021) "Digital Currency and Privacy," *Working Paper*, https://ssrn.com/abstract=3838718.

Keister, Todd and Cyril Monnet (2020) "Central bank digital currency: Stability and information," *Working paper, Rutgers University and University of Bern*.

Lee, Michael and Rodney Garratt (2021) "Monetizing Privacy," *FRB of New York Staff Report* (958).

Meaning, Jack, Ben Dyson, James Barker, and Emily Clayton (2018) "Broadening narrow money: Monetary policy with a central bank digital currency," Bank of England Staff Working Paper No. 724, May.

Monnet, Cyril., Asgerdur. Petursdottir, and Mariana Rojas-Breu (2021) "Central Bank Account for All: Efficiency and Risk Taking," *Working Paper*.

Parlour, Christine A, Uday Rajan, and Haoxiang Zhu (2020) "When fintech competes for payment flows," *Available at SSRN 3544981*.

Piazzesi, Monika and Martin Schneider (2020) "Credit lines, bank deposits or CBDC? competition and efficiency in modern payment systems," *Unpublished, Stanford University*.

Williamson, Stephen D. (2021) "Central bank digital currency and flight to safety," *Journal of Economic Dynamics and Control*, 104146, https://doi.org/10.1016/j.jedc.2021.104146.

Appendix: Proofs

Proposition 1. *There is an unique interior equilibrium with* $\{\gamma_n^*, \rho_n^*\}$ *in the economy. If* $\bar{k}$ *increases,* γ_n^* *and* $\bar{\pi}_n$ *increase with a more liquid bank portfolio.*

Proof. The equation that pins down the optimal γ can be written as:

$$1 - \gamma = \frac{1}{2} - \frac{1}{2}\{\frac{1}{1+T(\gamma)}\}^2 \tag{28}$$

In which $T(\gamma) = (\frac{1}{\gamma} - 1)[C - \frac{1}{\delta}(1-\gamma)]$ and $C = \delta R - \delta k$. Denote the RHS of the equation above as $g_1(\gamma)$. To prove it is decreasing and concave in γ, I need to show $\frac{dT}{d\gamma} < 0$ and $\frac{d^2T}{d^2\gamma} > 0$. Take the first order:

$$\begin{aligned}\frac{dT(\gamma)}{d\gamma} &= -\frac{1}{\gamma^2}(C - \frac{1}{\delta} + \frac{1}{\delta}\gamma^2)\\ &< 0\end{aligned}$$

Since $C - \frac{1}{\delta} > 0$ by assumption (positive bank return). The second order:

$$\begin{aligned}\frac{d^2T(\gamma)}{d^2\gamma} &= \frac{2}{\gamma^3}(C - \frac{1}{\delta} + \frac{1}{\delta}\gamma^2) - \frac{1}{\gamma^2}\frac{2\gamma}{\delta}\\ &= \frac{2}{\gamma^3}[C - \frac{1}{\delta} + \frac{\gamma^2}{\delta} - \frac{\gamma^2}{\delta}]\\ &= \frac{2}{\gamma^3}[C - \frac{1}{\delta}]\\ &> 0\end{aligned}$$

Thus the RHS function $g_1(\gamma)$ is concave. In addition, $G(\gamma \to 0) = \frac{1}{2}$ and $G(1) = 0$. When $\gamma = 1$, the slope $\frac{\partial g_1(\gamma)}{\partial \gamma} < -1$. Thus, the RHS is concave in γ and there is a single crossing interior point $\gamma^* \in (0,1)$ that solves the equation. □

Lemma 1. *As marginal information loss* β *is higher, a larger fraction of borrowers will stay using debit card payment. The equilibrium management cost* k_{dc}^* *increases.*

Proof. The equilibrium fraction of debit card payment user $\bar{b}^*$ can be expressed as:

$$\begin{aligned}\bar{b}^* &= \frac{\beta + (\frac{1}{\delta} - 1)\delta\bar{k}}{\beta + (\frac{1}{\delta} - 1)}\\ &= 1 - \frac{(\frac{1}{\delta} - 1)(1 - \delta\bar{k})}{\beta + (\frac{1}{\delta} - 1)}\end{aligned}$$

It can be shown that $\frac{d\bar{b}^*}{d\beta} > 0$. In addition, the equilibrium CBDC management cost $k_{dc}^* =$

$\bar{k} + \beta(1 - \bar{b}^*)$. Take the derivative to β:

$$\begin{aligned}
\frac{dk_{dc}^*}{d\beta} &= (1 - \bar{b}^*) - \beta \frac{d\bar{b}^*}{d\beta} \\
&= \frac{(\frac{1}{\delta} - 1)(1 - \delta\bar{k})}{\beta + (\frac{1}{\delta} - 1)} - \frac{\beta}{\beta + (\frac{1}{\delta} - 1)} \cdot \frac{(\frac{1}{\delta} - 1)(1 - \delta\bar{k})}{\beta + (\frac{1}{\delta} - 1)} \\
&= \frac{(\frac{1}{\delta} - 1)^2(1 - \delta)\bar{k}}{\beta + (\frac{1}{\delta} - 1)} \\
&> 0
\end{aligned}$$

So that as β increases, both $\bar{b}^*$ and k_{dc}^* increase at the equilibrium. □

Proposition 5. *As β_p decreases, k_{dc}^* and $\bar{b}^*$ both decrease. The CBDC rate ρ_0^* unambiguously decreases. The debit card rate ρ_1^* decreases if $\beta_p < \bar{A}$; and increases otherwise.*

Proof. Note the additional risk-sharing

$$\begin{aligned}
\Delta H &= -(k_{dc}^* - \delta\bar{k})(1 - \bar{b}^*) \\
&= -[(1 - \delta)\bar{k} + \beta(1 - \bar{b}^*)] \cdot (1 - \bar{b}^*)
\end{aligned}$$

It can be expressed as a function of CBDC payment user $(1 - \bar{b}^*)$. Since $\bar{b}^*$ is increasing in β, it can be seen there are competing effects in ΔH change. CBDC payment users shrink (fraction $(1 - \bar{b}^*)$ decreases), while the per cost $(k_{dc}^* - \delta\bar{k})$ increase.

$$\begin{aligned}
\frac{d\Delta H}{d\beta} &= (1 - \bar{b}^*)^2 - [(1 - \delta)\bar{k} + 2\beta(1 - \bar{b}^*)] \cdot \frac{d\bar{b}^*}{d\beta} \\
&= \frac{(\frac{1}{\delta} - 1)(1 - \delta\bar{k})}{[\beta + (\frac{1}{\delta} - 1)]^2} \cdot \{(\frac{1}{\delta} - 1) - 2(1 - \delta)\bar{k} - \frac{2(\frac{1}{\delta} - 1)(1 - \delta\bar{k})}{1 + \frac{1}{\beta}(\frac{1}{\delta} - 1)}\}
\end{aligned}$$

Note as β increases, the terms in the large brackets decrease. And $\frac{d\Delta H}{d\beta} > 0$ when $\beta \to 0$. The threshold can be found when the term equals zero, $\bar{A} = (1 - 2\delta\bar{k})(\frac{1}{\delta} - 1)$. For $\beta < \bar{A}$, $|\Delta H|$ increase in β and corresponds to more costs from risk-sharing.

□